# 3D Rotation-Invariant Description from Tensor Operation on Spherical HOG Field

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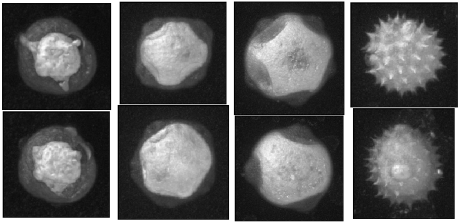
bioss CENTRE FOR BIOLOGICAL SIGNALLING STUDIES

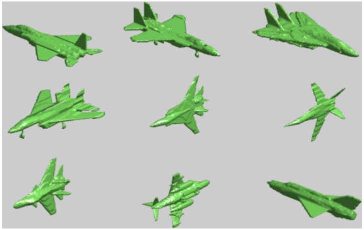
1. Motivations and Proposal
2. HOG as Continuous Angular Signal
3. Regional Description
4. Experiment and Application
5. Conclusion

# Motivations

- Rotational-invariance is important for many applications with 3D volumetric data.

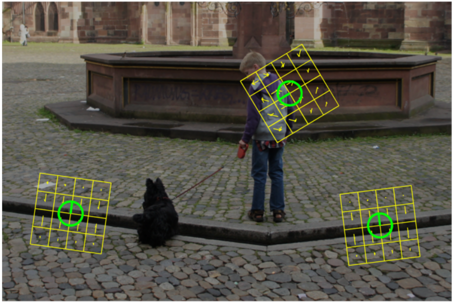


3D microscopic images of pollen

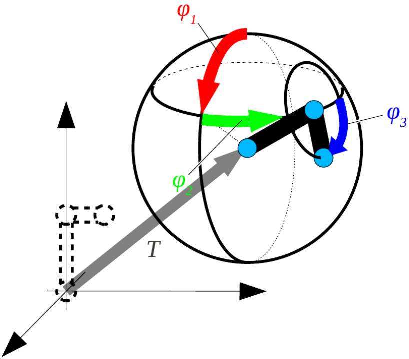


3D shape models

# Rotation-invariance from pose normalization



Pose normalization in 2D SIFT




3D pose is more complicated

- 2D → 3D: 2 more angles to be determined
- Pose normalization becomes more complicated and less reliable

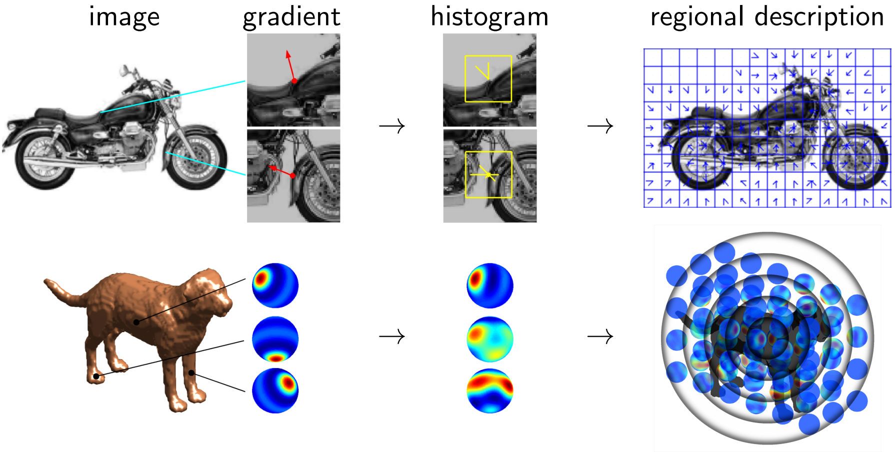
# Rotation-invariance from Fourier analysis

- Spherical Harmonics → Analytical rotational-invariance

  [Q. Wang, *et al*, Rotational Invariance based on Fourier Analysis in Polar and Spherical Coordinates. IEEE Transactions on PAMI, 2009]

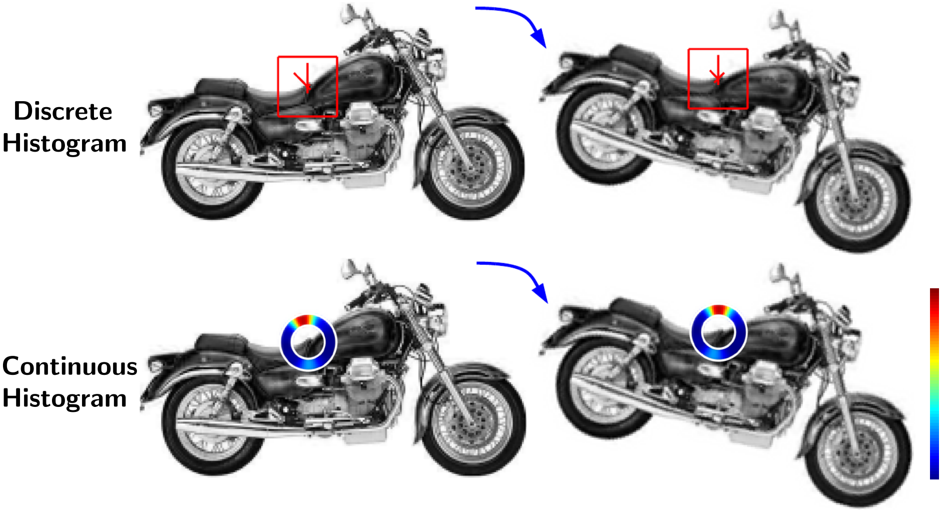
- Our contribution:
  HOG + Spherical Harmonics → robust 3D rotation-invariant descriptions

# Proposal: Spherical HOG Feature + Regional Description




using Spherical Harmonics for features in the spherical coordinates

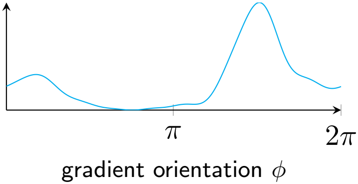
# 2D HOG as continuous circular signals




- Rotation can be easily addressed in Fourier space.

# The continuous histogram in Fourier space

:



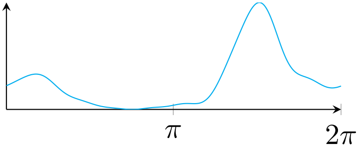

Fourier basis $e^{\mathbf{i}n\phi}$

Fourier coefficients

$$c_n = \frac{1}{2\pi}\int_0^{2\pi} f(\phi) e^{-\mathbf{i}n\phi}\, d\phi$$

# The continuous histogram in Fourier space

 :




gradient orientation $\phi$

Fourier basis $e^{\mathbf{i}n\phi}$

Fourier coefficients

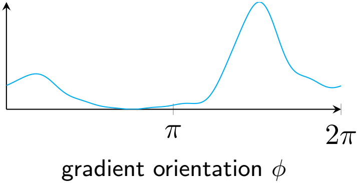
$$c_n = \frac{1}{2\pi}\int_0^{2\pi} f(\phi)e^{-\mathbf{i}n\phi}\,d\phi$$

 $= c_0 + c_1 e^{i\phi} + c_2 e^{i2\phi} + \ldots$

$Fourier($  $) = [c_0, c_1, c_2 \cdots]$

# The continuous histogram in Fourier space

 :




Fourier basis $e^{\mathbf{i}n\phi}$

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 $= c_0 + c_1 e^{i\phi} + c_2 e^{i2\phi} + \ldots$

$Fourier($  $) = [c_0, c_1, c_2 \cdots]$

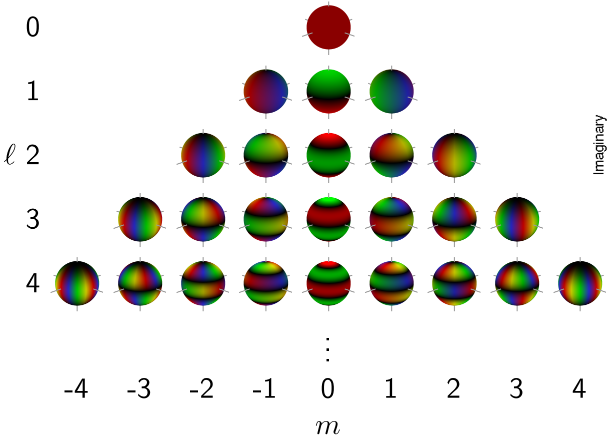
Under a rotation (of angle $\theta$),  $\rightarrow$ 

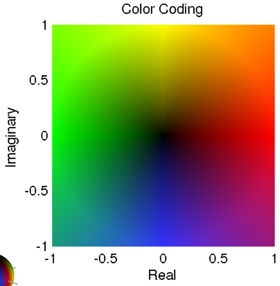
$Fourier($  $) = [c'_0, c'_1, c'_2 \cdots]$

$$c'_n = e^{-\mathbf{i}n\theta} c_n$$

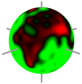
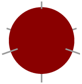
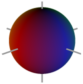
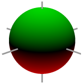
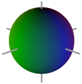
# Circles → Spheres, Fourier basis → Spherical Harmonics

Analogously to the Fourier basis $e^{\mathbf{i}n\phi}$, the wave functions on a sphere are called spherical harmonics



# Expansion on spheres → Spherical Harmonic Coefficients

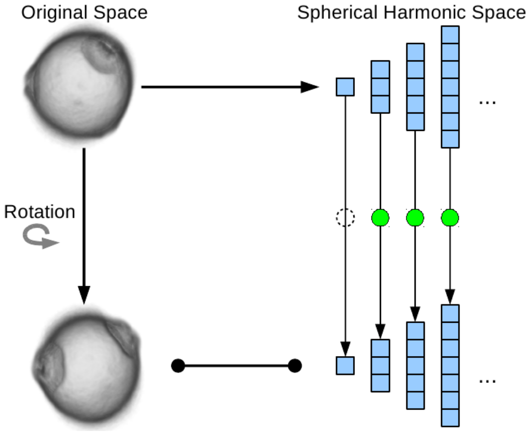
 $= \hat{f}^0_0 \cdot$  $+ \hat{f}^1_{-1} \cdot$  $+ \hat{f}^1_0 \cdot$  $+ \hat{f}^1_1 \cdot$  $+ \cdots$

$\hat{f}^\ell_m$ are complex-valued coefficients:

$$\underbrace{\hat{f}^0_0,}_{\text{band 0: } \hat{\mathbf{f}}^0} \quad \underbrace{\hat{f}^1_{-1}, \hat{f}^1_0, \hat{f}^1_1,}_{\text{band 1: } \hat{\mathbf{f}}^1} \quad \underbrace{\hat{f}^2_{-2}, \hat{f}^2_{-1}, \hat{f}^2_0, \hat{f}^2_1, \hat{f}^2_2,}_{\text{band 2: } \hat{\mathbf{f}}^2} \quad \cdots$$

The coefficients in the same band transform together under rotations.

# Rotation in Spherical Harmonic Space

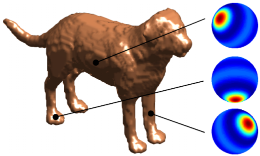



$\hat{\mathbf{f}}^{\ell}$

Multiplying the coefficients with Wigner-D Matrices $\mathbf{D}^{\ell}(\mathfrak{g})$
($\mathfrak{g}$ represents the rotation)

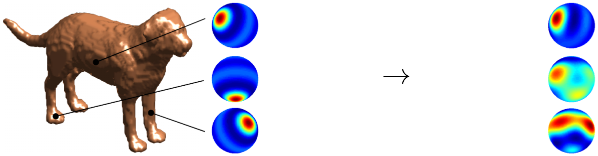
$\mathbf{D}^{\ell}(\mathfrak{g})\hat{\mathbf{f}}^{\ell}$

# 3D HOG represented in Spherical Harmonic space



- Take an individual gradient as a Dirac function on sphere
- Project it onto Spherical Harmonics

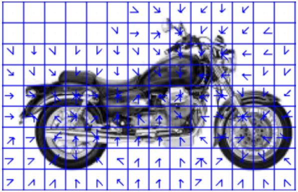
# 3D HOG represented in Spherical Harmonic space



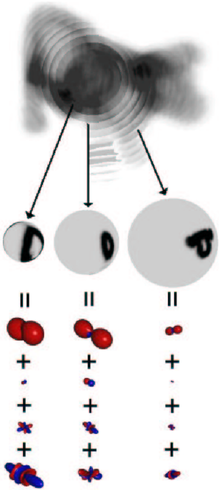
- Take an individual gradient as a Dirac function on sphere
- Project it onto Spherical Harmonics

- Spatial aggregation $\rightarrow$ spatial smoothing on spherical harmonic coefficients

# Regional description of HOG arrangement



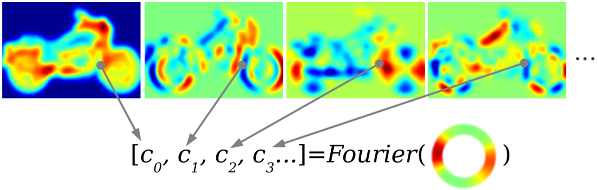
grid sampling



shell sampling + expansion

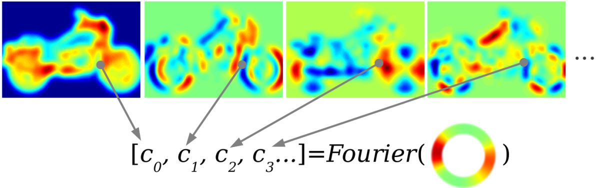
M. Kazhdan, *et al*, Rotation invariant spherical harmonic representation of 3D shape descriptors, 2003.

# 2D example: radial sampling + Fourier expansion

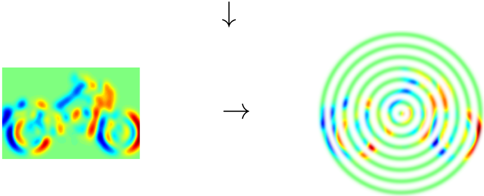



pixel-wise HOG
in Fourier space

# 2D example: radial sampling + Fourier expansion

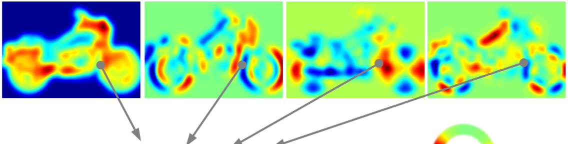



pixel-wise HOG
in Fourier space



radial sampling
(in each channel)

# 2D example: radial sampling + Fourier expansion

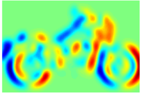
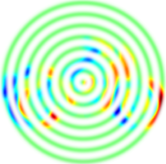


...

pixel-wise HOG
in Fourier space

$[c_0, c_1, c_2, c_3 \ldots] = Fourier($  $)$

↓

 → 

radial sampling
(in each channel)

↓

$Fourier($  $) = [cc_0, cc_1, cc_2 \cdots]$

$\{cc_n\}$ has simple rotation behaviour

Fourier expansion
(at each sampled radius)

# How to create rotation-invariance

| | | |
|---|---|---|
| image | $f$ | $f' =$ rotate $f$ by angle $\theta$ |
| feature | $a$ | $a' = e^{-\mathbf{i}m\theta}a$ |
| another feature | $b$ | $b' = e^{-\mathbf{i}n\theta}b$ |

# How to create rotation-invariance

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|---|---|---|
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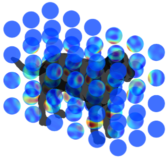
| feature | $a$ | $a' = e^{-\mathbf{i}m\theta}a$ |
| another feature | $b$ | $b' = e^{-\mathbf{i}n\theta}b$ |
| energy | $\|\|a\|\|^2 = \overline{a}a$ | $\|\|a'\|\|^2 = \overline{a}e^{\mathbf{i}m\theta}e^{-\mathbf{i}m\theta}a = \|\|a\|\|^2$ |
| coupled value | $\overline{a}b$ | $\overline{a'}b' = e^{\mathbf{i}(m-n)\theta}\overline{a}b$ |

Energy is rotation-invariant.

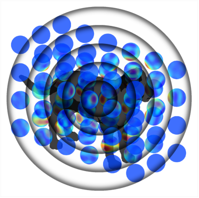
$\overline{a}b$ is rotation-invariant if $m = n$.

# Solution for the 3D HOG Field



Dense HOG feature in *SH* space

→



shell sampling

→

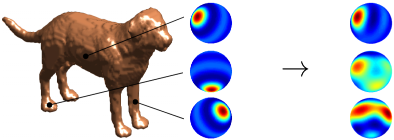
Spherical Tensorial Expansion

- The dense HOG features in Spherical Harmonics space need the Spherical Tensorial expansion.

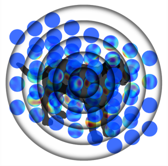
  [M. Reisert and H. Burkhardt. Spherical tensor calculus for local adaptive filtering, 2009 ]

- Rotation-invariance: coupling two expansion coefficients which transform with the same Wigner-D matrices.

# Summary of approach



- Representing HOG in Spherical Harmonics space

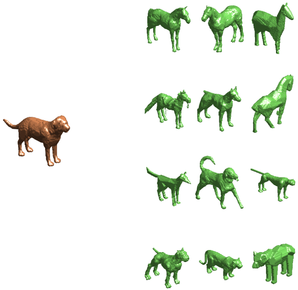


- Describing local region by shell-sampling + expansion
- Coupling the output of the same rotation behaviour

# Evaluation on Princeton Shape Benchmark

| Method | Nearest Neighbour(%) | First Tier(%) | Second Tier(%) | E-measure(%) | DCG(%) |
|---|---|---|---|---|---|
| HOG-ST | **67.4** | **37.4** | **47.6** | **28.0** | **63.8** |
| SH | 56.0 | 28.4 | 37.6 | 22.3 | 56.0 |
| StrT-ST | 61.7 | 30.7 | 39.6 | 23.2 | 58.2 |
| $\text{BoF}_{SHcorr}$ | 62.4 | / | / | / | / |
| $\text{HOG}_{align}$ | 58 | 27 | 35 | 21 | 55 |

- Using the evaluation tools from the benchmark



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| BoF$_{SHcorr}$ | 62.4 | / | / | / | / |
| HOG$_{align}$ | 58 | 27 | 35 | 21 | 55 |

- HOG-ST: Spherical HOG + shell-wise tensorial expansion.

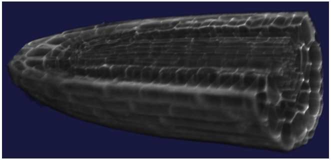
- SH: Spherical Harmonics Descriptor. [P. Shilane, *et al*, 2004.]
- StrT-ST: Structure Tensor + shell-wise tensorial expansion. [H. Skibbe, *et al*, 2009. ]
- BoF$_{SHcorr}$: Bag-of-features approach with Spherical Correlation for feature comparison. [J. Fehr, *et al*, 2009. ]
- HOG$_{align}$: HOG features on pose-normalized 3D shapes. [M. Scherer, *et al*, 2010. ]

# SHREC 2009 Generic Shape Benchmark

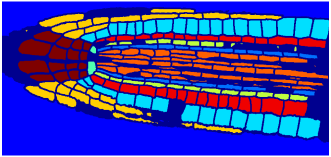
| Method | Nearest Neighbour(%) | First Tier(%) | Second Tier(%) | E-measure(%) | DCG(%) |
|---|---|---|---|---|---|
| HOG-ST | **90.0** | **50.6** | **62.0** | **43.4** | **80.2** |
| StrT-ST | 81.2 | 39.0 | 49.3 | 34.1 | 71.2 |
| $\text{HOG}_{align}$ | 75 | 41 | 52 | 35 | 71 |

- HOG-ST: Spherical HOG + shell-wise tensorial expansion.
- StrT-ST: Structure Tensor + shell-wise tensorial expansion. [H. Skibbe, *et al*, 2009. ]
- $\text{HOG}_{align}$: HOG features on pose-normalized 3D shapes. [M. Scherer, *et al*, 2010. ]

# Application on Biological data



Raw data



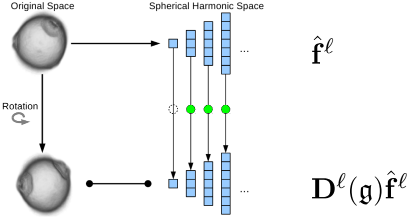
One slice of labelled data

- Data: confocal microscopic imaging of plant roots
- Target: assign voxels into different classes (background / cell-wall / 6 layers...)
- Voxel-wise rotation-invariant descriptions + SVM

# Fast computation for voxel-wise descriptions

- Shell-wise expansion is not efficient for dense description.
- Spherical Gaussian Derivative (*SGD*) is an efficient alternative.

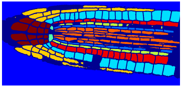
  [M. Reisert and H. Burkhardt. Spherical tensor calculus for local adaptive filtering, 2009 ]
- It keeps the simple rotation behaviour.



# Multi-scale SGD Filtering on HOG fields

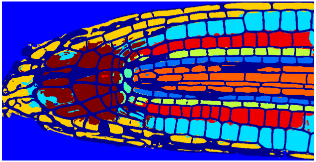
14 out of 240 energy features

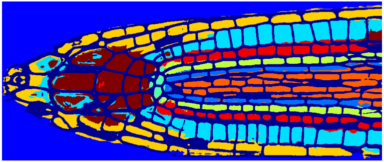
# Experiment result



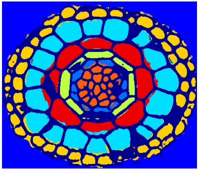
Train SVMs with the ground-truth labels

Apply to other roots:

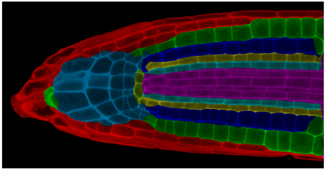




classification result



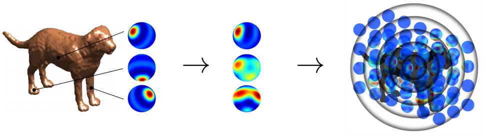
a cross-section



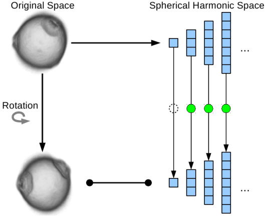
refined region segmentation (by energy minimization)

# Conclusion

- A robust 3D rotation-invariant description is proposed, based on HOG and Spherical Harmonics.



- Relating features and operations to Fourier basis (2D) and Spherical Harmonics (3D) can lead to simple rotation behaviour.




$$\hat{\mathbf{f}}^{\ell}$$

$$\mathbf{D}^{\ell}(\mathfrak{g})\hat{\mathbf{f}}^{\ell}$$

Thank you!