2D/3D Rotation-Invariant Detection using Equivariant Filters and Kernel Weighted Mapping

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Overview

- Equivariance from Fourier analysis

  - using features which have simple multiplicative transforms under rotations, instead of purely invariant features.

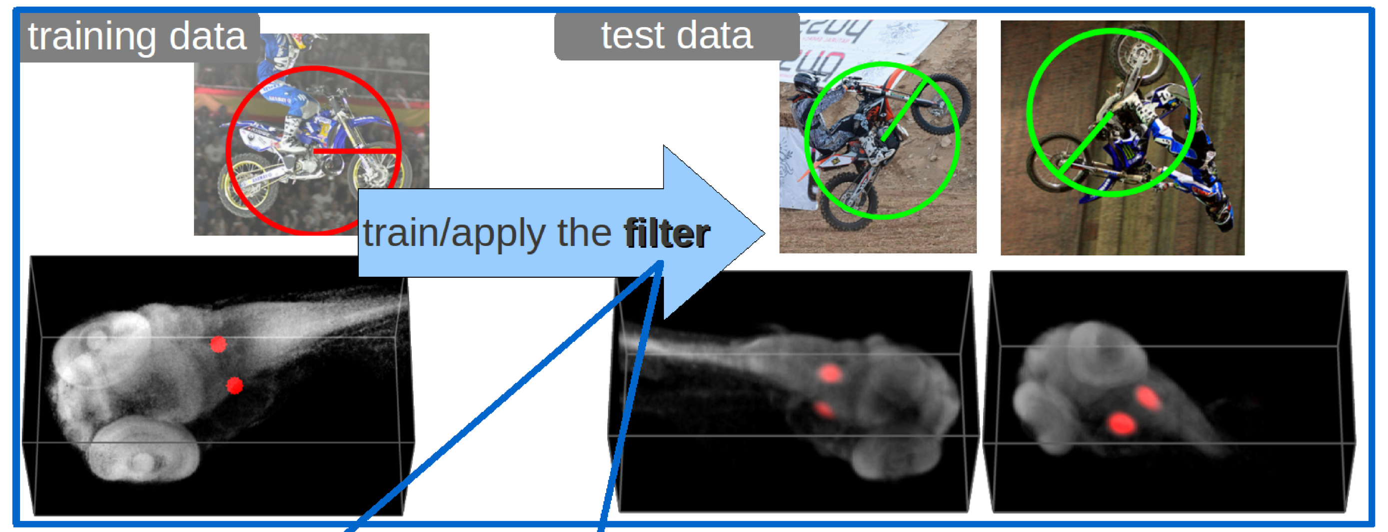
  - the desired "rotation-invariance" is analytically guaranteed.

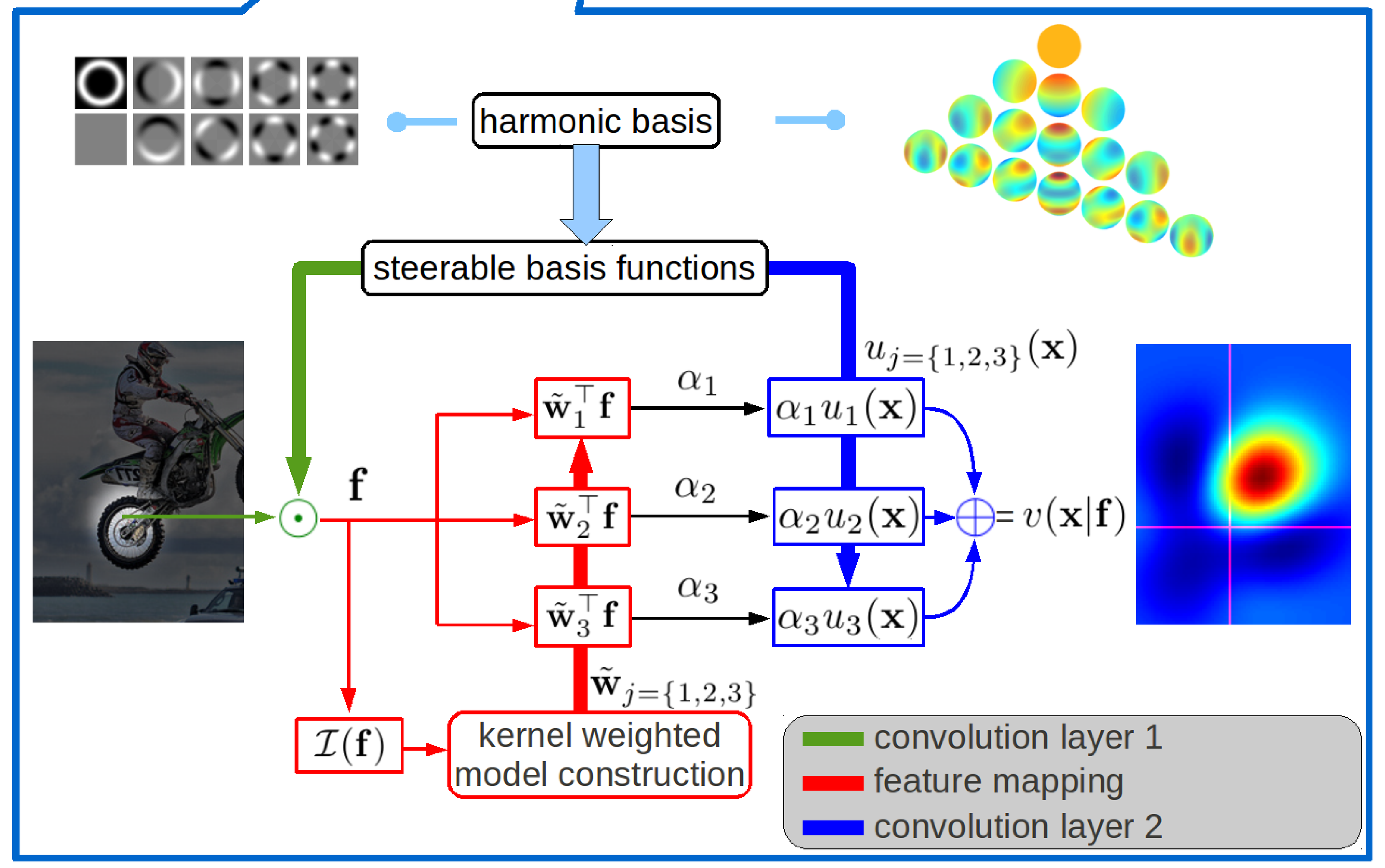
- A flexible non-linear model utilizing covariant/invariant features together

  - the core feature mapping function is constructed for each feature vector by a kernel weighted interpolation.

- Easy generalization from 2D images to 3D volumetric data

  - analogous analytic forms.





Equivariance from Fourier Analysis

- The desired "invariance" in detection problems is the **equivariance** [1].

- In the polar coordinates (r, φ), with an arbitrary radial profile $R(r)$, a basis function $u = R(r)e^{im\varphi}$ has the "self-steerability" as

$$u(r, \varphi - \beta) = e^{-im\beta} u(r, \varphi). \tag{1}$$

- Considering a convolution $H(I) = I * u$, we have

$$\underbrace{H(I(r, \varphi - \beta))}_{\text{filtering on the rotated image}} = e^{im\beta} \underbrace{[H(I)](r, \varphi - \beta)}_{\text{rotated filtering output of the original image}}, \tag{2}$$

$$H_2(H_1(I(r, \varphi - \beta))) = e^{i(m_1+m_2)\beta}[H_2(H_1(I))](r, \varphi - \beta). \tag{3}$$

- $m_1 + m_2 = 0 \Leftrightarrow$ **equivariance**: $\mathcal{H}(I(r, \varphi - \beta)) = [\mathcal{H}(I)](r, \varphi - \beta)$.

Building the Feature Mapping

- For the modeling capacity, we need a nonlinear feature mapping between the two layers of convolutions (Eq.(3)). **This mapping has to respect the equivariance.**

$$\text{image} \xrightarrow{*\,u} \mathrm{f} \xrightarrow{\text{mapping}} \alpha \xrightarrow{*\,u} \text{output}$$

- A rotation-invariant kernel function as similarity measure:

$$\mathcal{K}_{\mathcal{I}}(\mathrm{f}, \mathrm{f}') = K(\mathcal{I}(\mathrm{f}), \mathcal{I}(\mathrm{f}')), \tag{4}$$

where $\mathcal{I}$ is an operator to create a rotation-invariant feature vector from given covariant features, K is a RBF kernel.

- Kernel weighted mapping

$$\alpha = \tilde{\mathrm{w}}^\top \mathrm{f} = \left[\frac{\sum_k (\mathcal{K}_{\mathcal{I}}(\mathrm{f}_k, \mathrm{f})\mathrm{w}_k)}{\sum_k \mathcal{K}_{\mathcal{I}}(\mathrm{f}_k, \mathrm{f})}\right]^\top \mathrm{f}, \tag{5}$$

where $\mathrm{f}_{k=\{1,\dots,k_{\max}\}}$ is a set of (selected) points distributed in the feature space, w_k (the local linear model at f_k) are the parameters to estimate.

From 2D to 3D

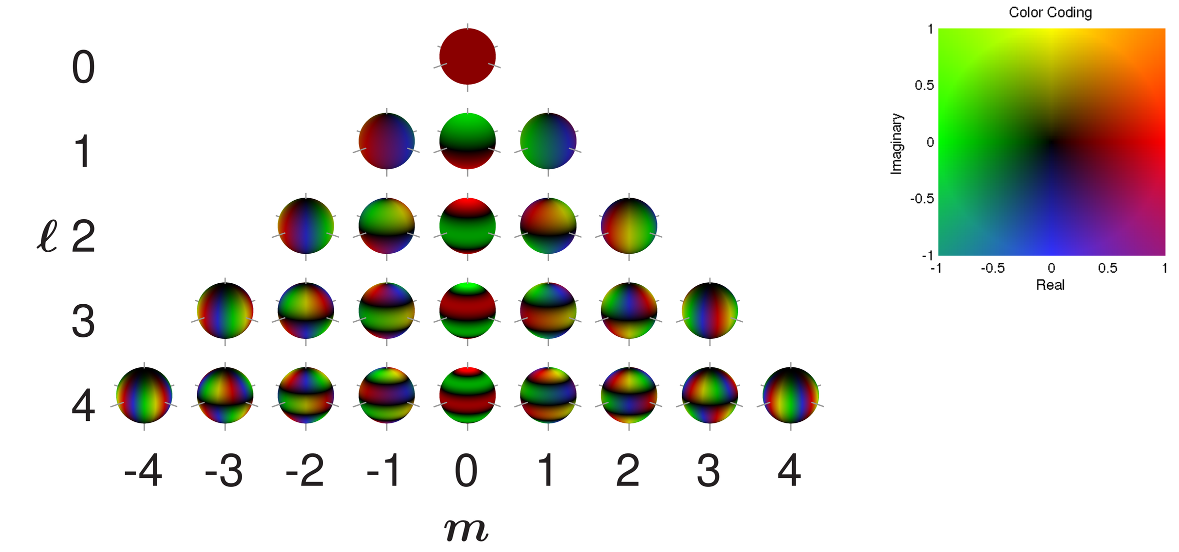
- Projection of a 2D angular signal on the Fourier basis

$$= \sum_m c_m e^{im\varphi} = c_0 \cdot \; + c_1 \cdot \; + c_2 \cdot \; + c_3 \cdot \; + \cdots$$

Under a rotation (of angle θ), $\rightarrow$

$$\hat{c}_0 = c_0, \quad \hat{c}_1 = e^{-i\theta} c_1, \quad \hat{c}_2 = e^{-i2\theta} c_2 \cdots$$

- **Spherical Harmonics**: the wave basis on a sphere (the angular part of the spherical coordinates)



- Projection of a 3D angular signal on the spherical harmonics

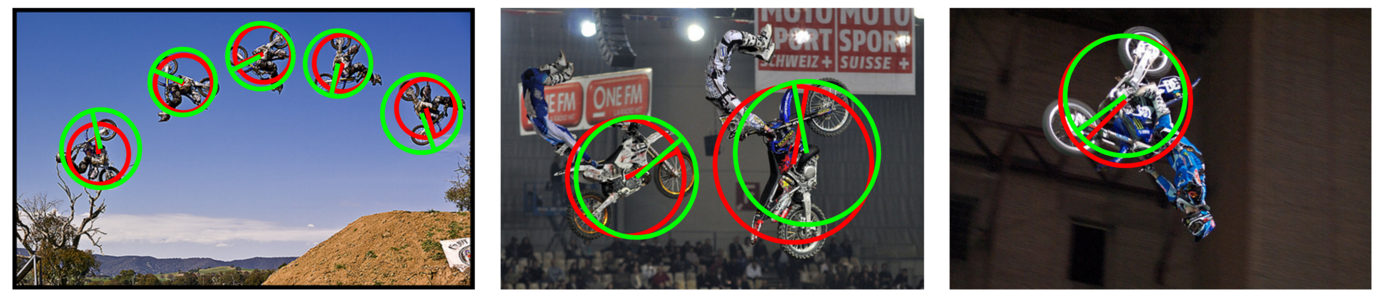
$$= c_0^0 \cdot \; + c_{-1}^1 \cdot \; + c_0^1 \cdot \; + c_1^1 \cdot \; + \cdots$$

Under an arbitrary 3D rotation $\mathbf{g}$, $\rightarrow$

$$\begin{aligned} \hat{c}_0^0 &= c_0^0 \\ [\hat{c}_{-1}^1, \hat{c}_0^1, \hat{c}_1^1]^\top &= \mathrm{D}^1(\mathbf{g})[c_{-1}^1, c_0^1, c_1^1]^\top \\ [\hat{c}_{-2}^2, \hat{c}_{-1}^2, \hat{c}_0^2, \hat{c}_1^2, \hat{c}_2^2]^\top &= \mathrm{D}^2(\mathbf{g})[c_{-2}^2, c_{-1}^2, c_0^2, c_1^2, c_2^2]^\top \\ &\cdots \end{aligned}$$

$D^\ell \in \mathbb{C}^{\ell \times \ell}$ is a matrix determined by the rotation angles [2].

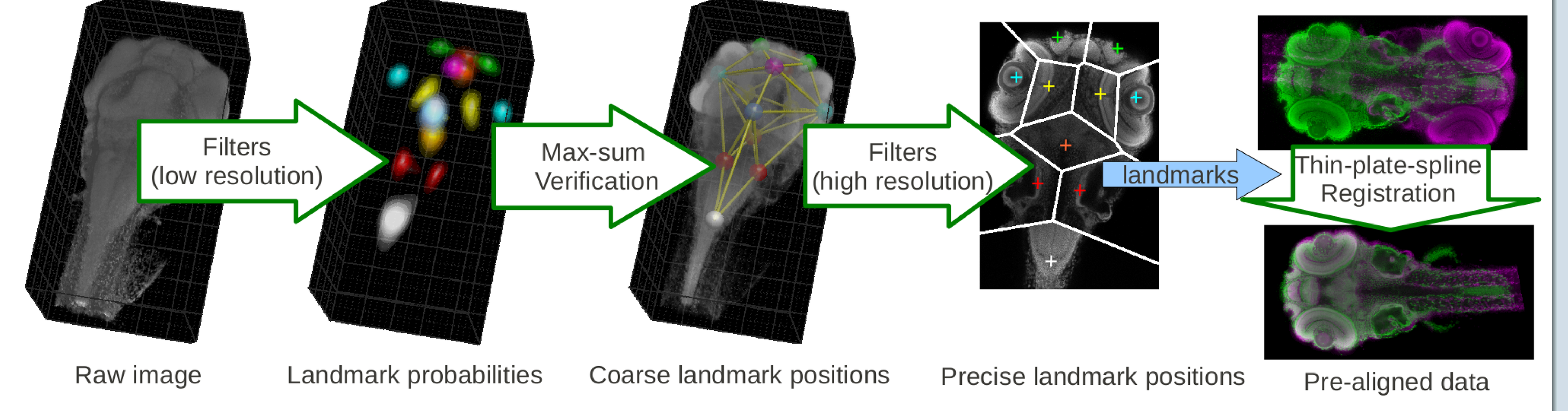
2D Experiment - Freestyle Motorbike



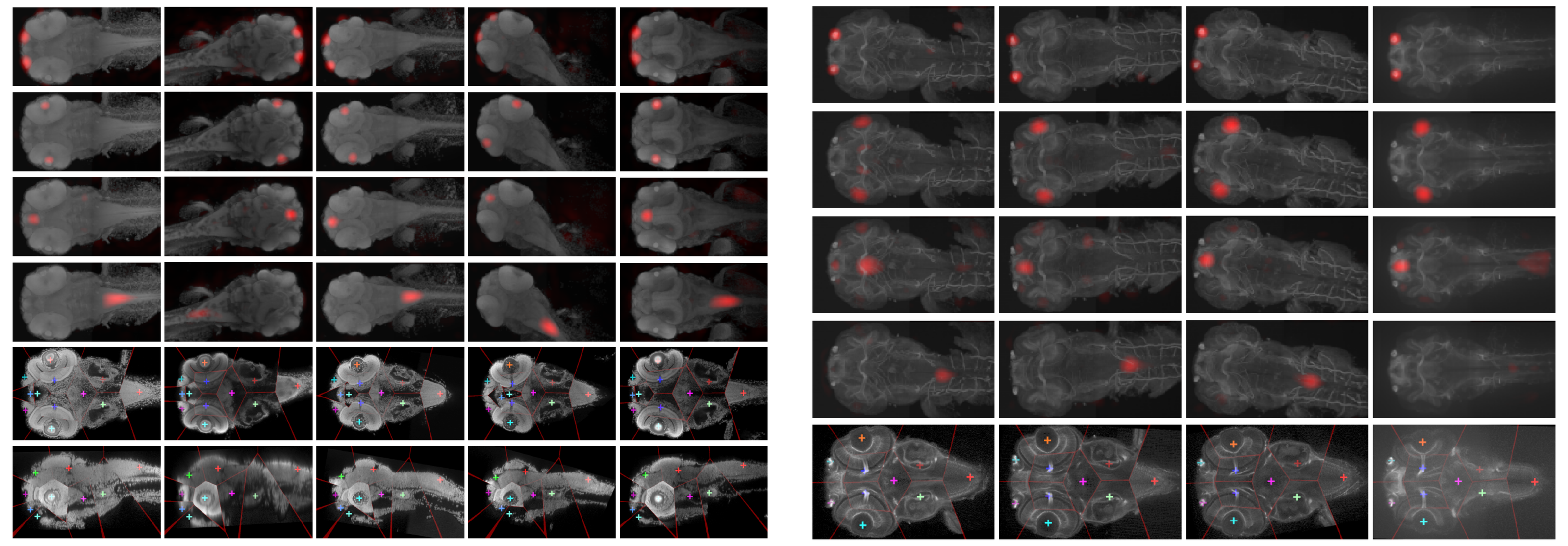
- Detections/ground-truth are drawn in green/red. The quantitative measure is comparable to [3].
- HOG features can be easily embedded into the method, as they can be represented as angular signals [4].

3D Landmark Detection in Zebrafish Embryos

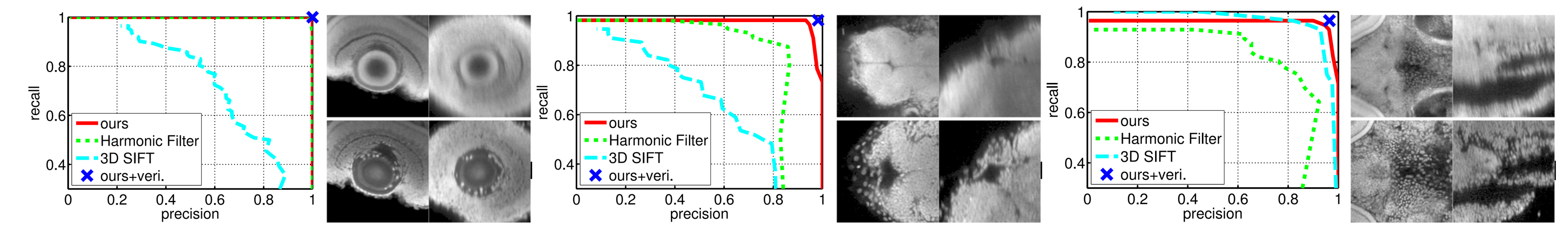
- Task: using landmark detection to initialize the elastic registration, for gene-expression colocalization analysis in zebrafish



- Qualitative result (nuclei/AcTub staining)



- Comparison against 3D SIFT and Harmonic Filter



- The full gene-expression colocalization analysis framework using this detection filter will appear in *Nature Methods* [5]. Implementation available on our website.

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