Chapter 4

Invariant Shape Characterization

Edge detection and description

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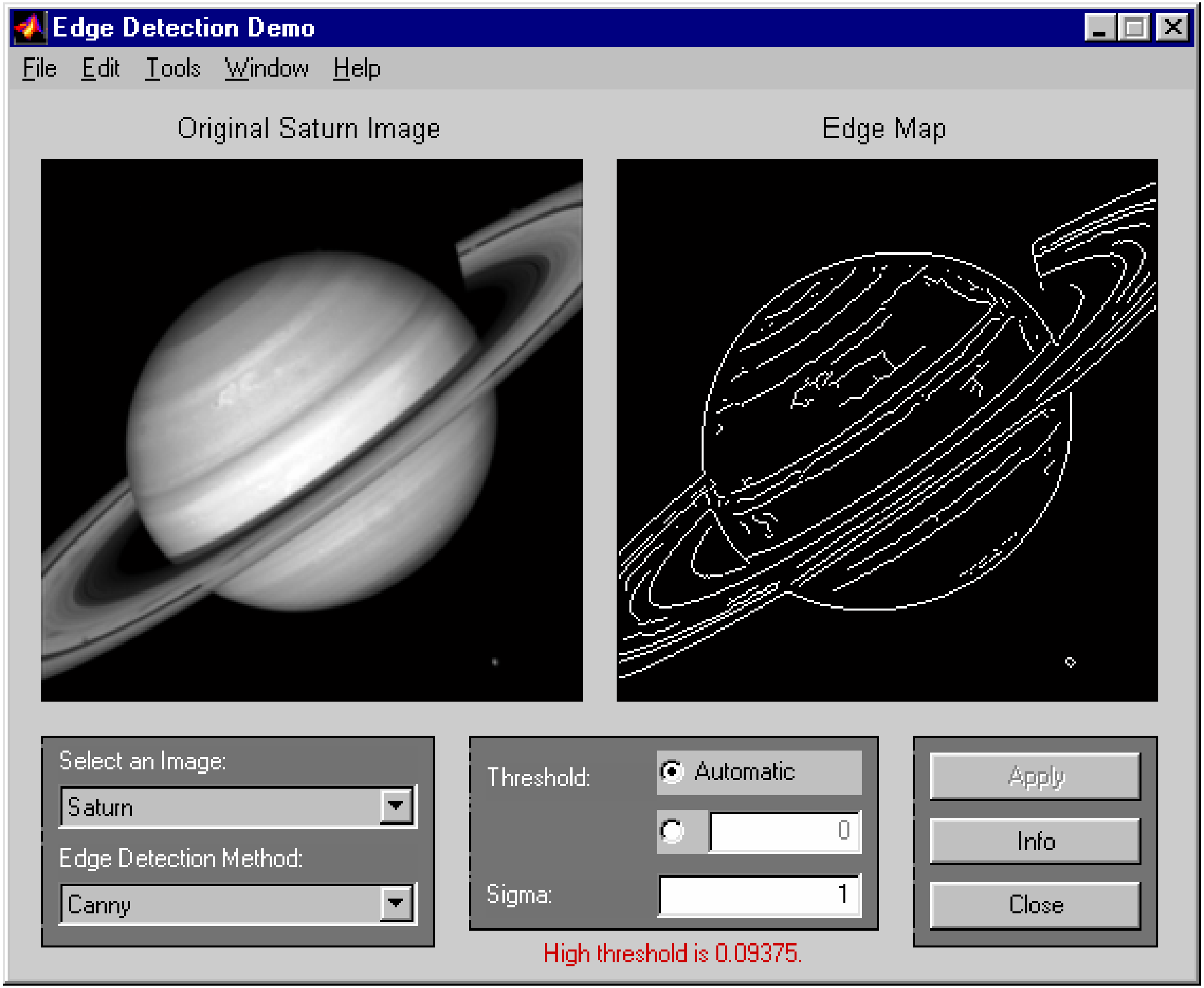
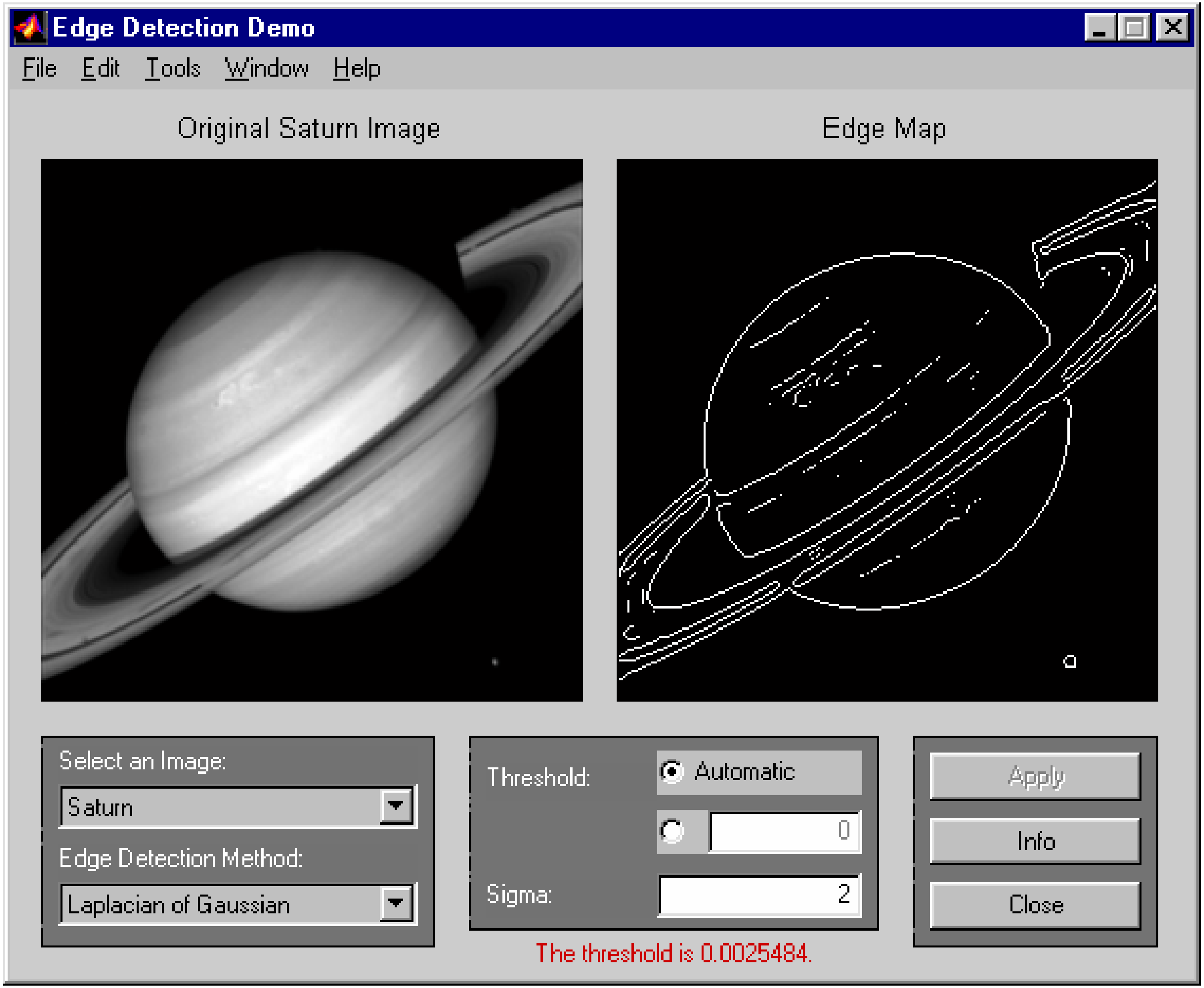
Edge filter “seagull”

Contour extraction using the Canny-operator

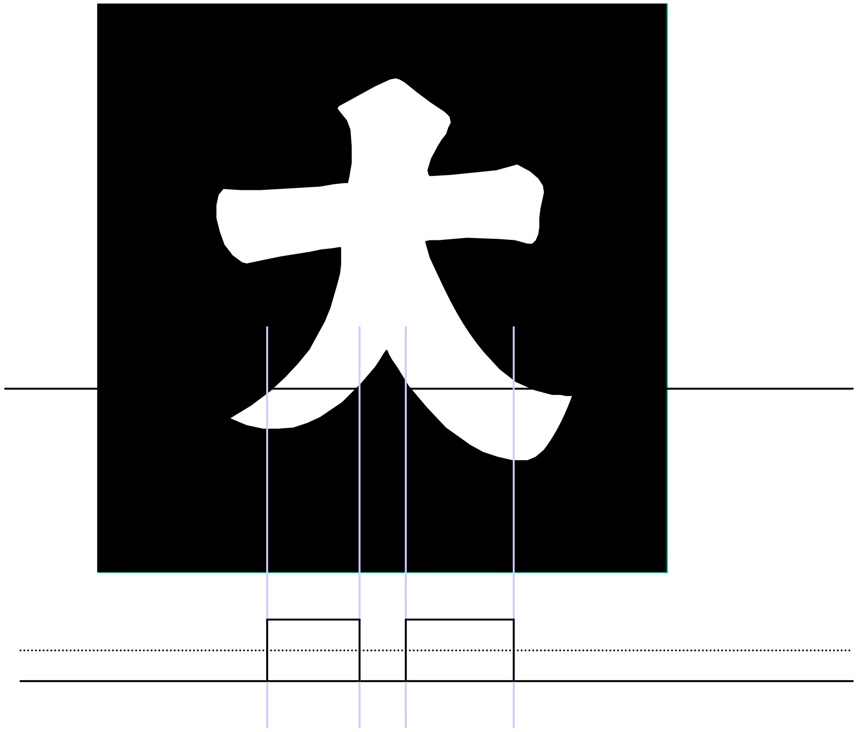


Contour extraction using the Laplacian of Gaussian



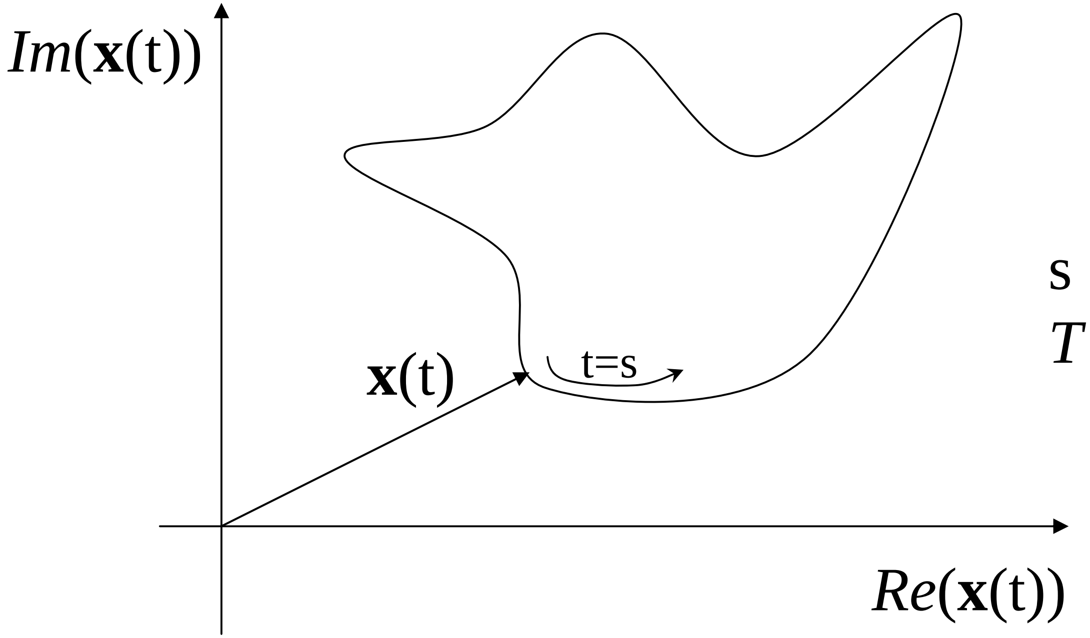
Contour extraction using simple threshold operation

For a well-defined contour the problem is significantly simpler.



Describing the edge in the complex plane

- Suppose that the edges of objects have already been extracted, which reduces the amount of data drastically!
- Closed edges form periodic functions in the complex plane, i.e. $\mathbf{x}(t+kT)=\mathbf{x}(t)$



s arc length

T perimeter

Representing closed edges using finite Fourier series with *N+1* complex coefficients

Periodic functions can be described using Fourier series. Let the class $\mathbb{F}^N$ denote the set of band-limited periodic patterns, which can be uniquely represented using the finite Fourier series with N+1 complex Fourier coefficients:

$$x(t) = \sum_{n=-N/2}^{n=+N/2} c_n e^{jn\omega t} \quad \text{with:} \;\; \omega = 2\pi / T$$

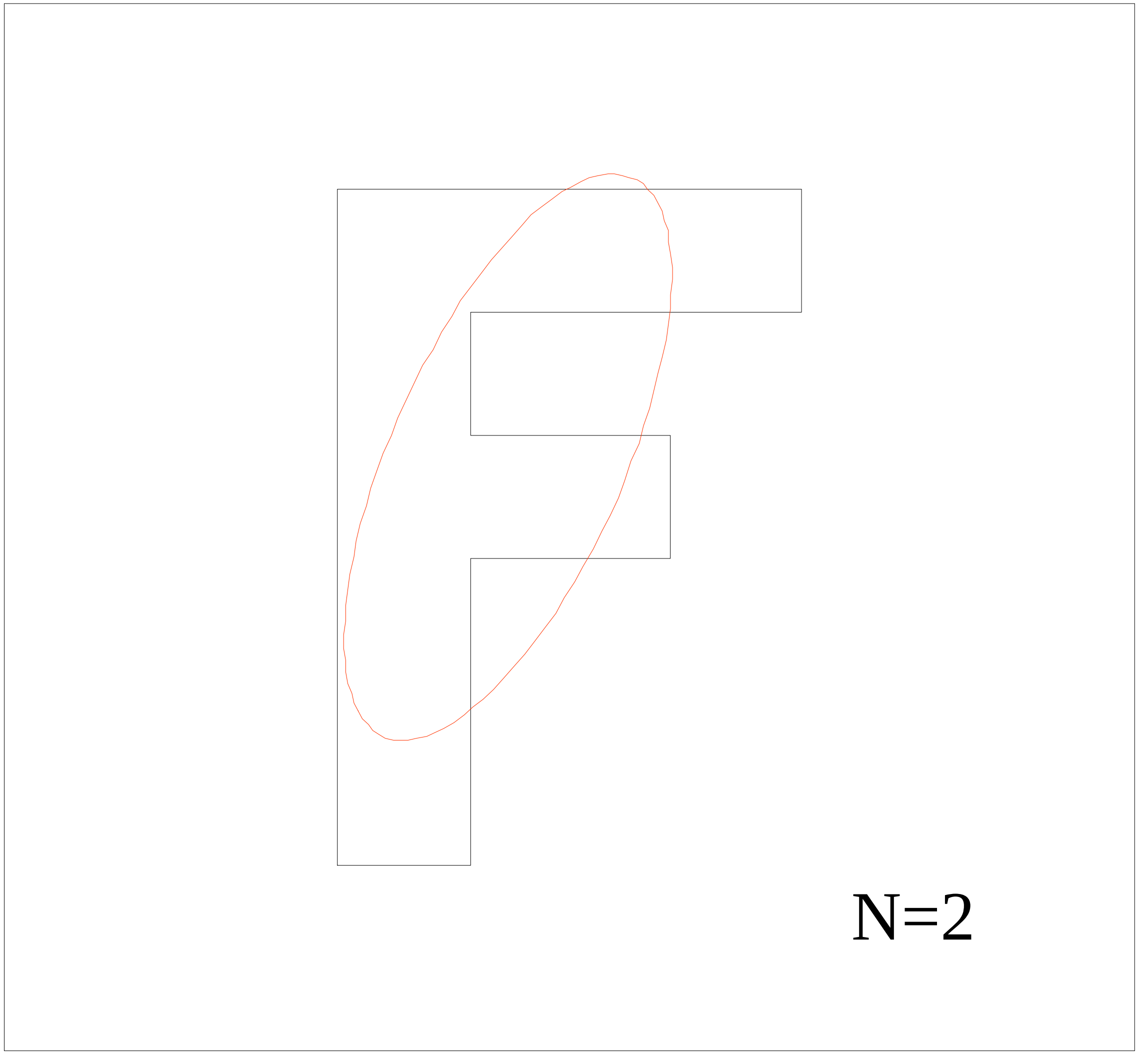
with the Fourier coefficients (FC):

$$c_n = \frac{1}{T}\int_0^T x(t) e^{-j2\pi nt/T} dt$$

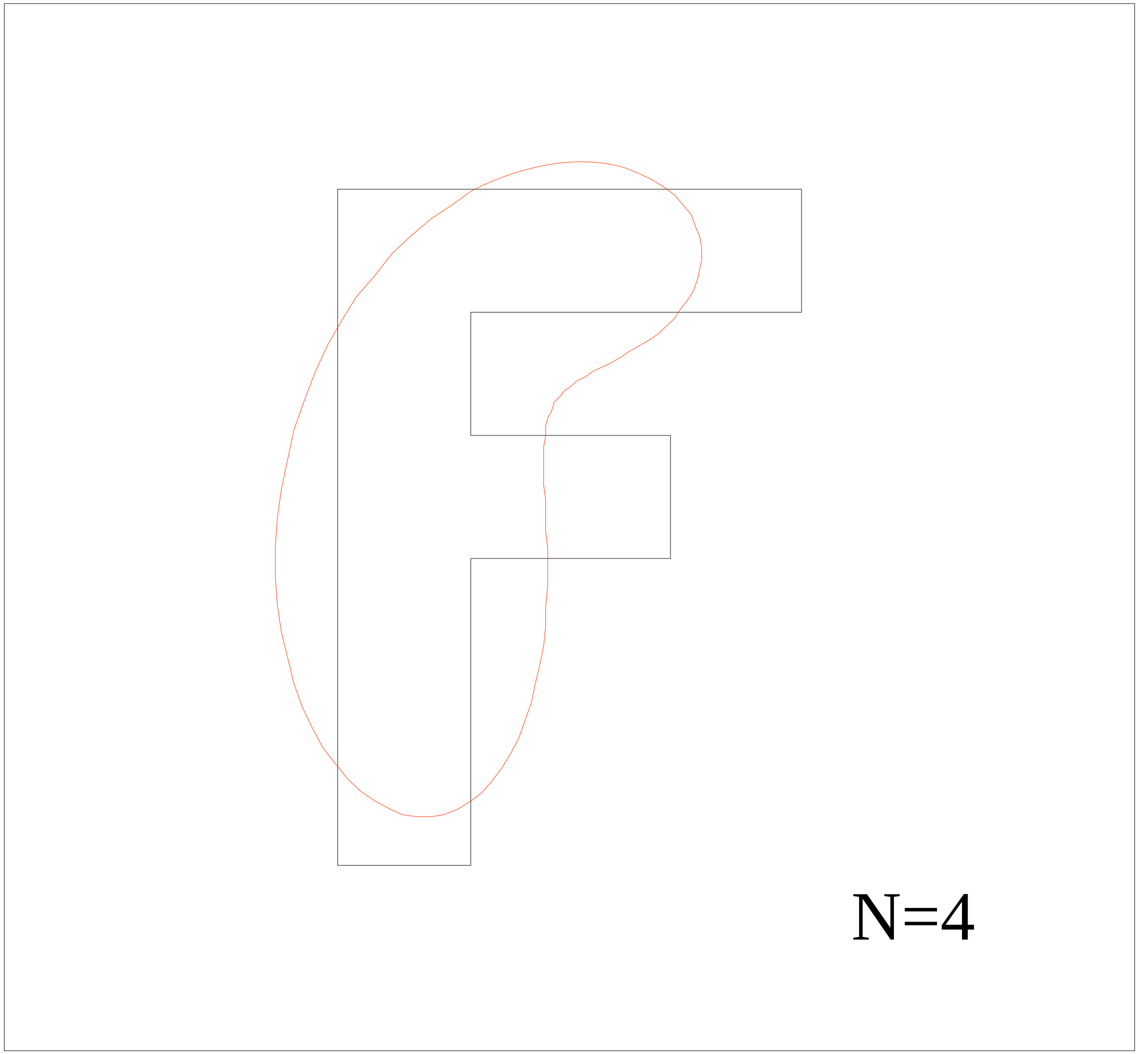
The direction of circulation is defined in a way, that with increasing arc length the inner region is located to the left of the contour line.

For complex periodic functions the FC are arbitrary, for real-valued functions applies: $c_{-n} = c_n^*$

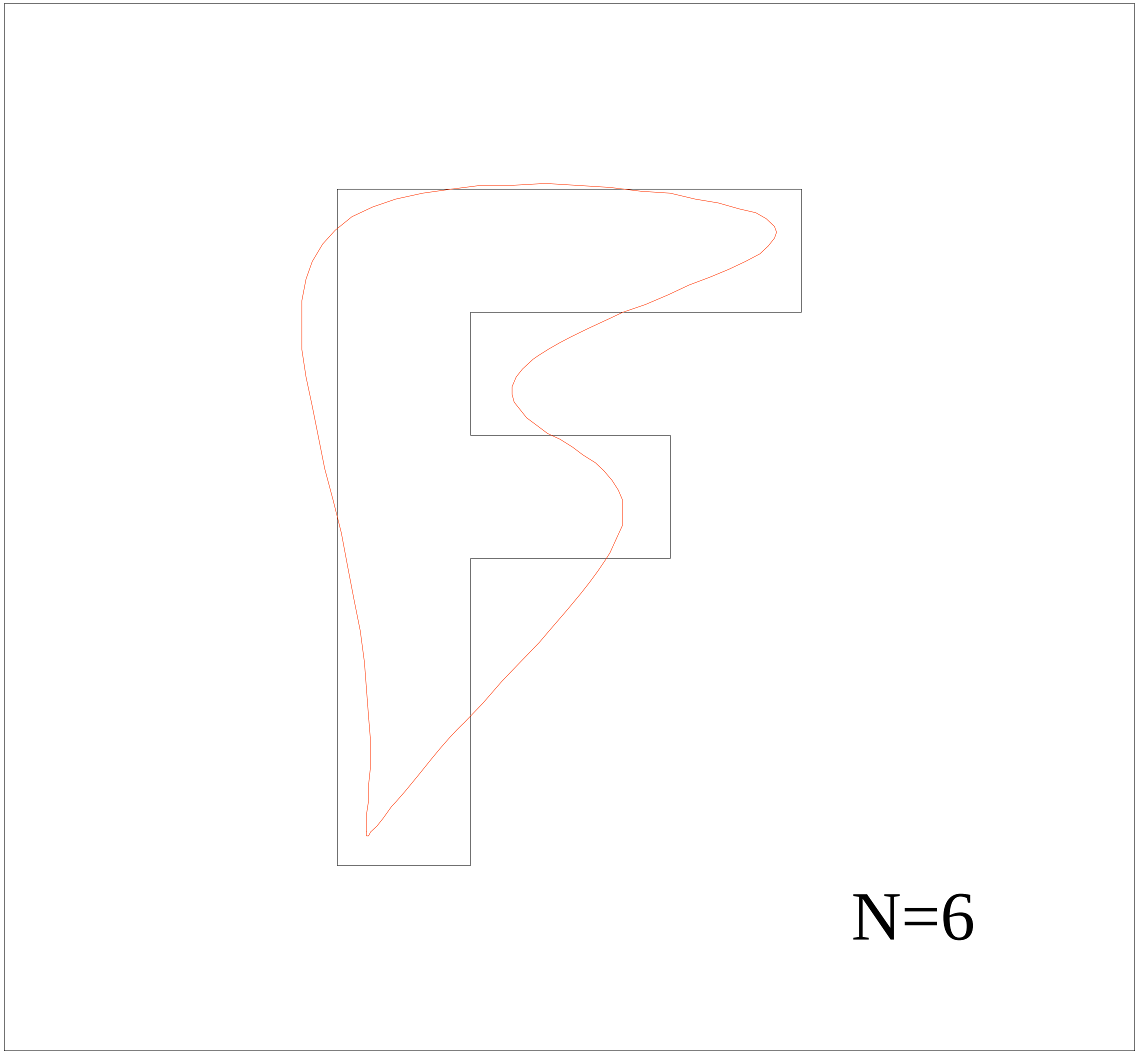
Fourier synthesis of a closed contour



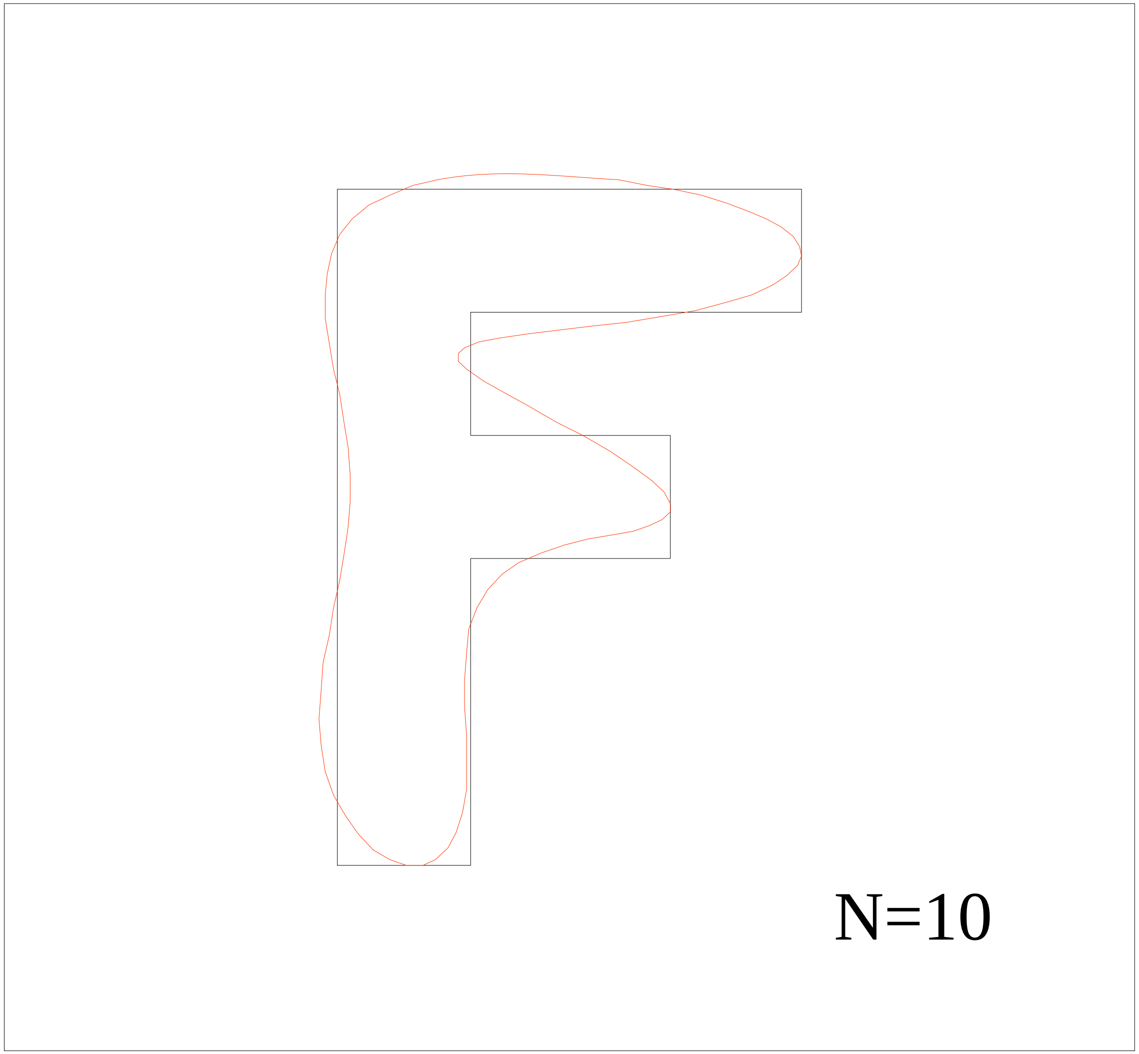
Fourier synthesis of a closed contour



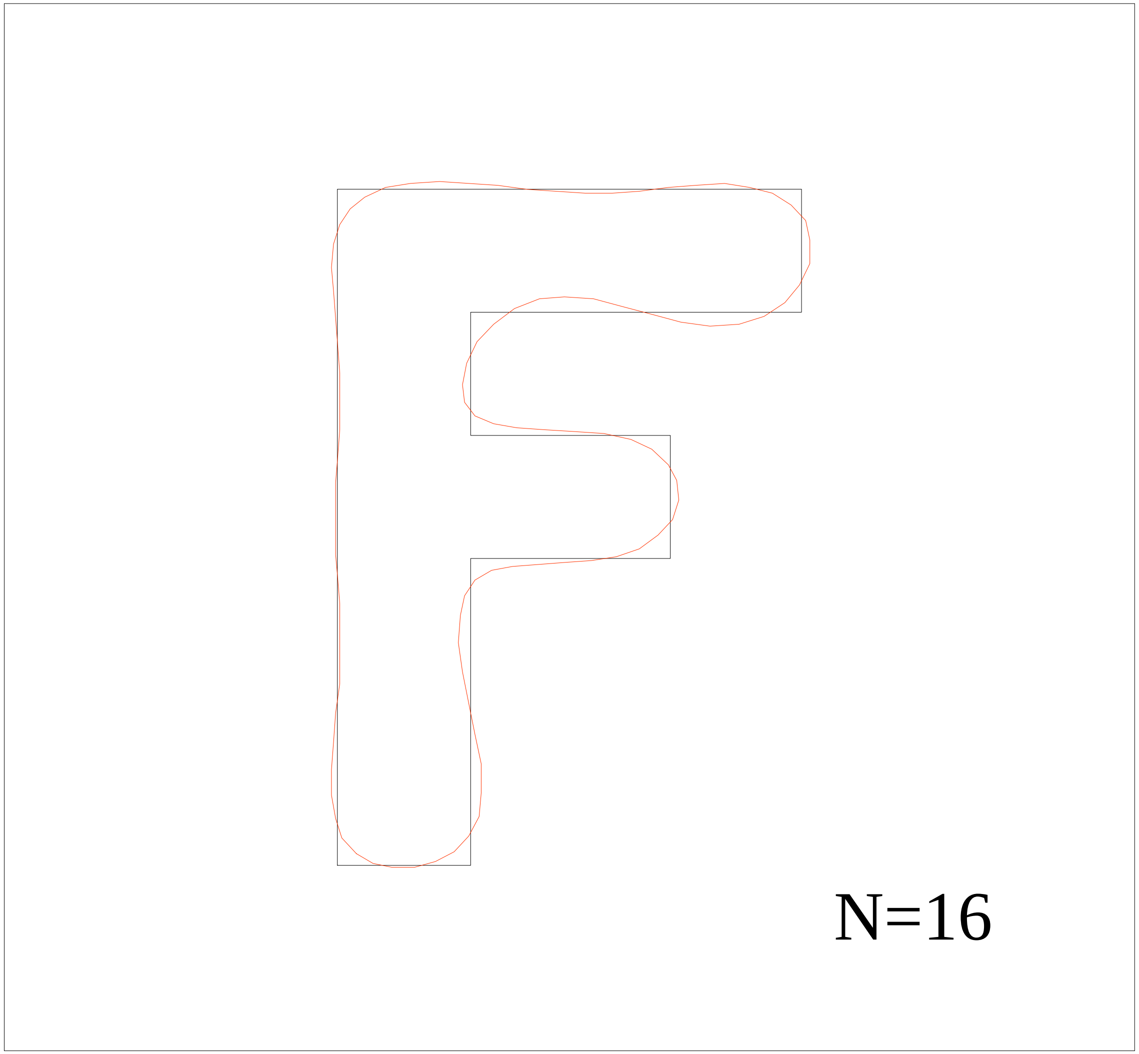
Fourier synthesis of a closed contour



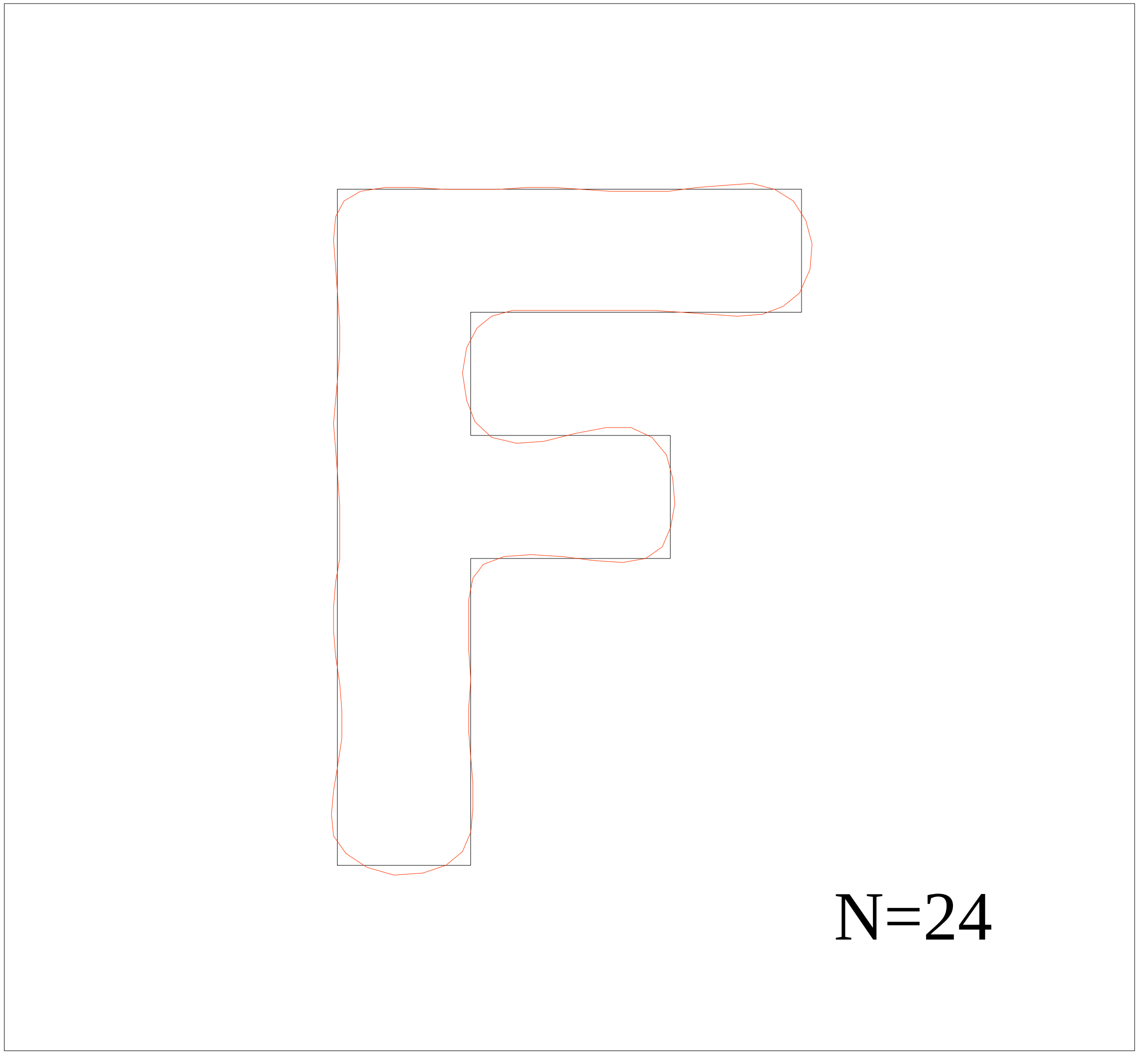
Fourier synthesis of a closed contour



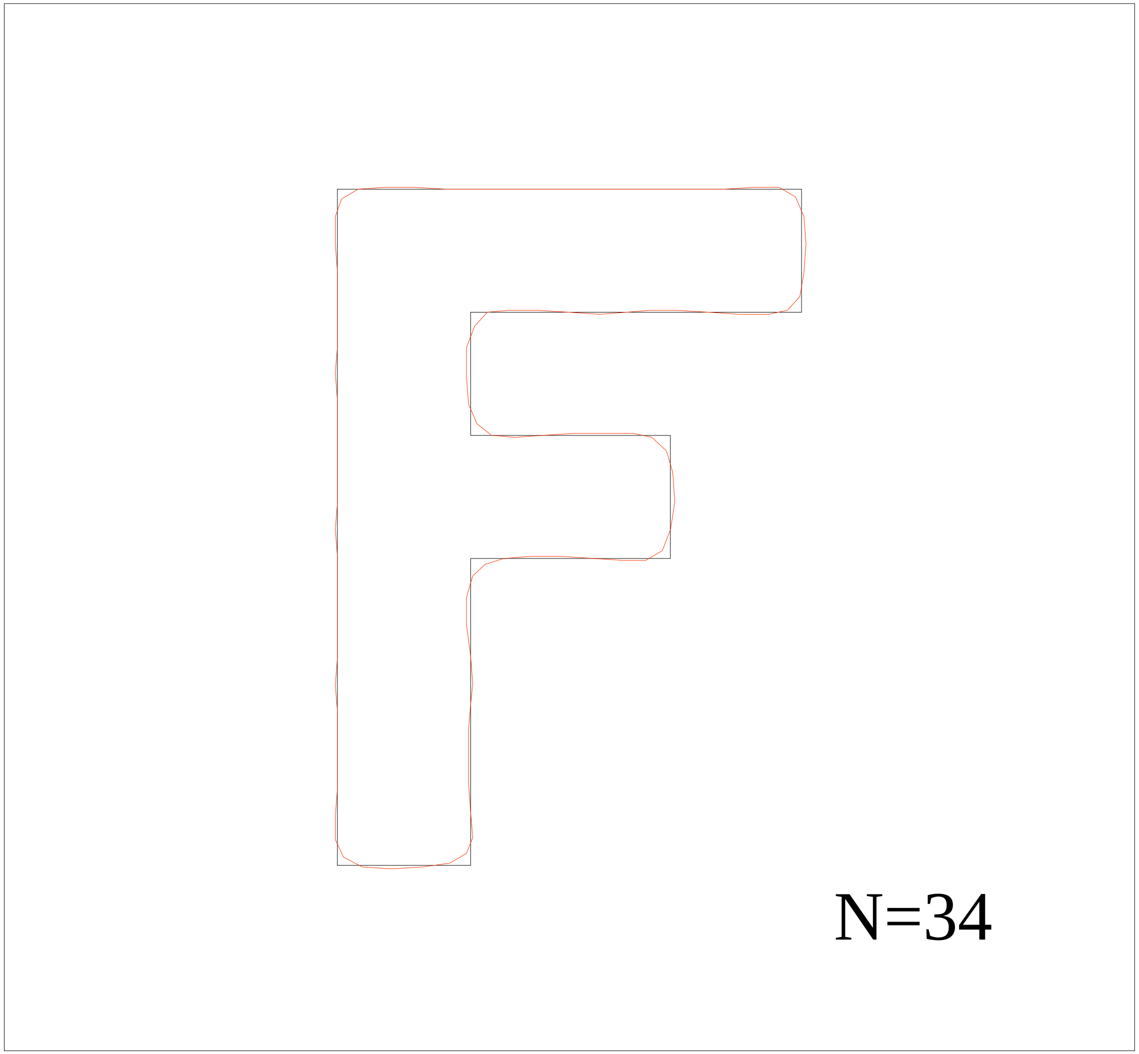
Fourier synthesis of a closed contour



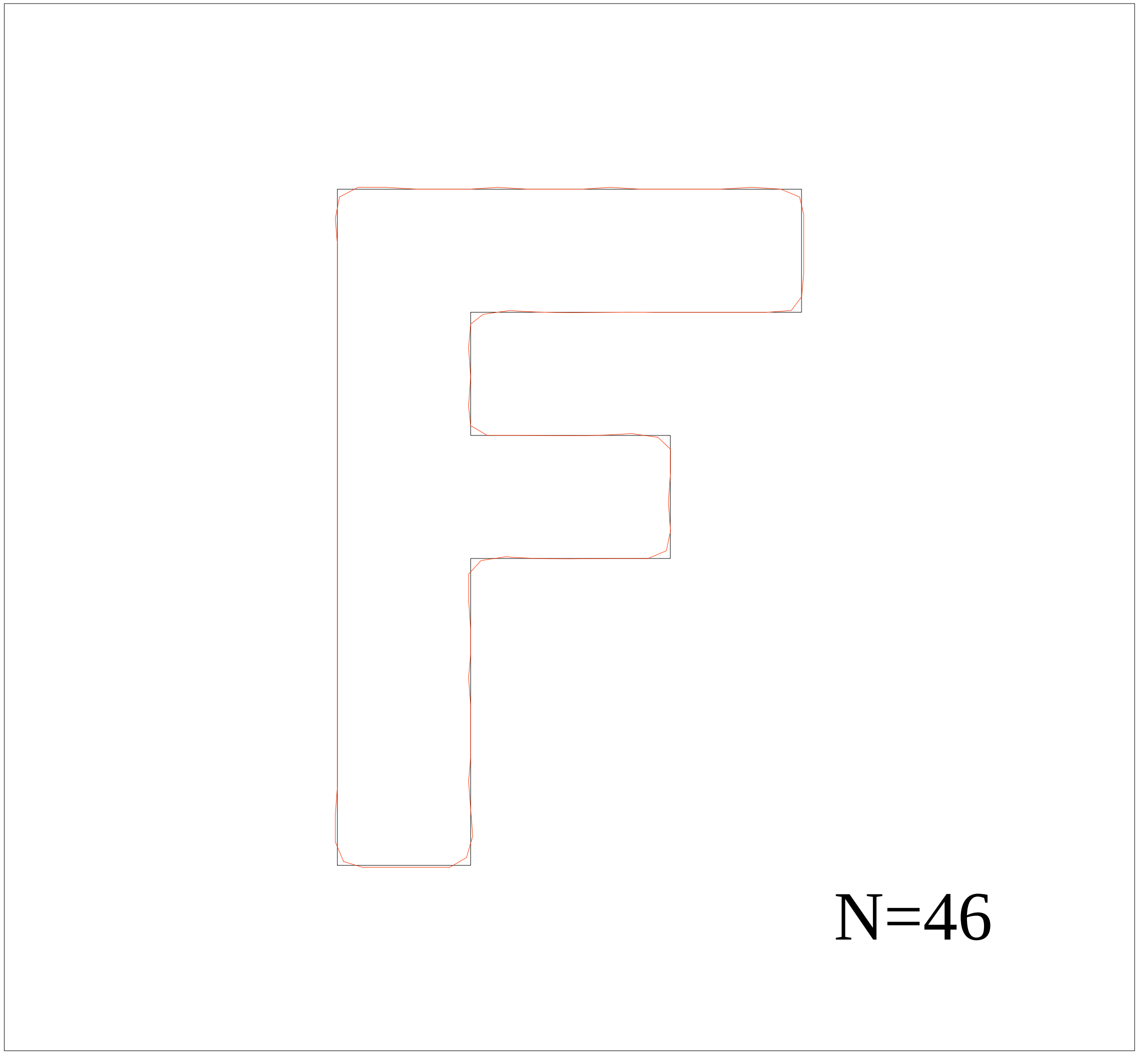
Fourier synthesis of a closed contour



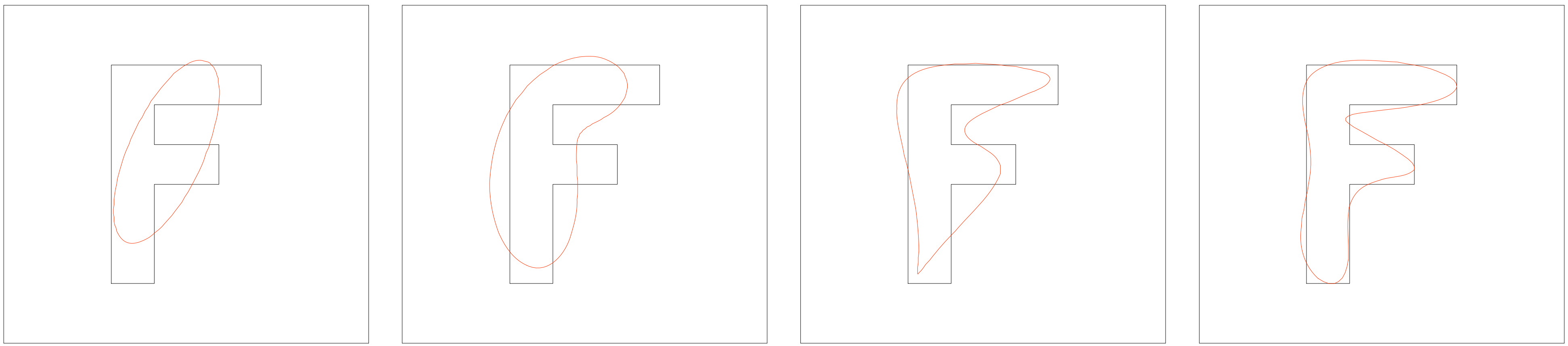
Fourier synthesis of a closed contour



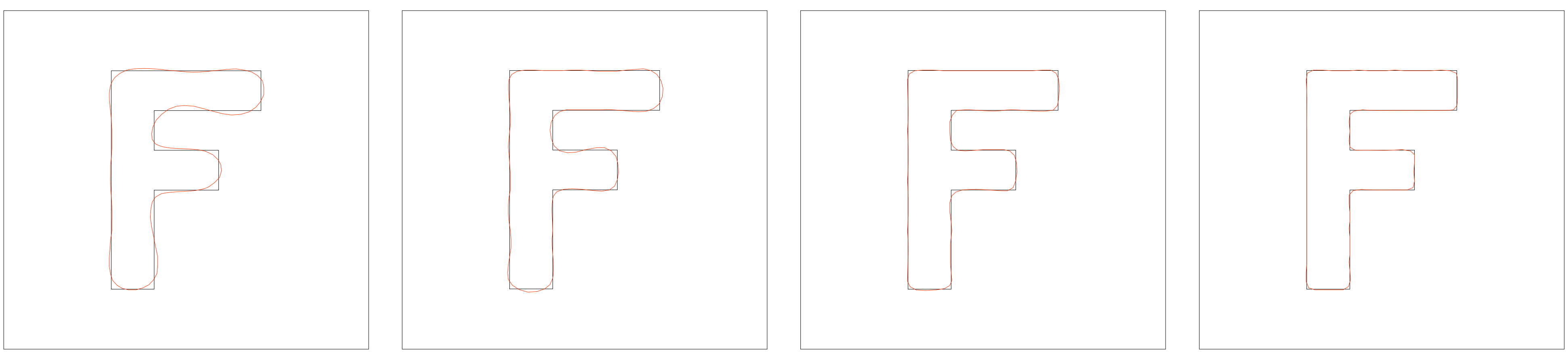
Fourier synthesis of a closed contour



Fourier synthesis of a closed contour



N=2 N=4 N=6 N=10

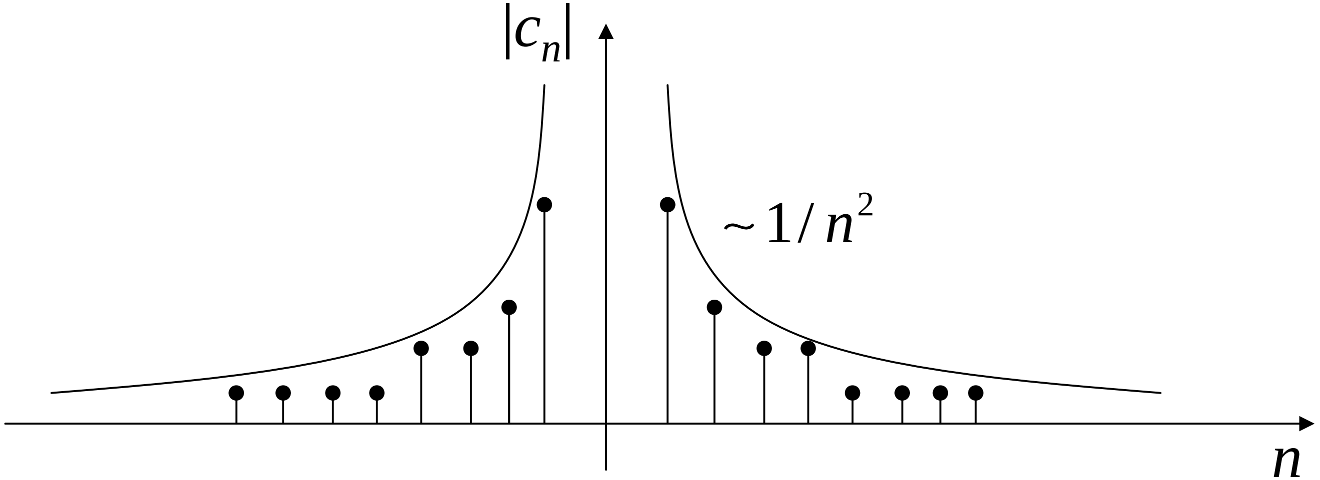


N=16 N=24 N=34 N=46

Properties of Fourier series

Representing a contour using a Fourier series has integral (global) character. Compared to representation with sample values in the original spectrum, the amount of data in the spectral region can be reduced drastically when the Fourier series is limited to few (low frequency) coefficients.

Closed contours are continuous and bounded. The magnitude of FC is proportional to $1/n^2$ and thus tends to 0.



Properties of Fourier series

The 0th FC c_0 indicates the location of the balance point of the line (contour line with constant mass allocation; wire frame graphics).

A Fourier series with only one coefficient represents a circle, which (depending on the frequency) can be traversed repeatedly. Together with the correspondig negative coefficient an ellipse results:

$$x(t) = c_0 + \underbrace{c_1 e^{j\omega t}}_{\text{circle, phasor running to left}} + \underbrace{c_{-1} e^{-j\omega t}}_{\text{circle, phasor running to right}}$$

c_n and c_{-n} represent an ellipse, which is traversed n-times.

Fourier descriptor demonstrationl

http://www.vision.ee.ethz.ch/~buc/brechbuehler/fourdem.html

The degrees of freedom

If the complex numbers c_0,c_1 und c_{-1} are chosen freely, 6 degrees of freedom result. An ellipse has only 5 degrees of freedom (2 translation, 2 axes and the rotation). Where remains the 6th degree of freedom?

Translating the starting point of the parameterization results in:

$$x(t+\alpha) = c_0 + c_1 e^{j\omega\alpha} e^{j\omega t} + c_{-1} e^{-j\omega\alpha} e^{-j\omega t}$$

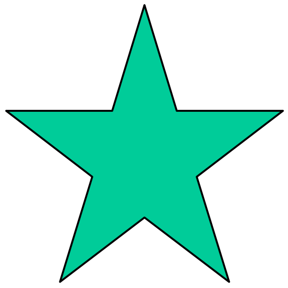
Choosing α to make $c_1 e^{j\omega\alpha}$ real, has the effect that one degree of freedom disappears. Thus the starting point is the 6th degree of freedom.

Rotation symmetry of degree *s*

An object is rotation symmetric of degree *s*, if it passes into itself when rotating about 360°/*s* .

Thus only FC with distance *s* can have values different from 0, namley for indices:

$$n = 1 \pm ks \quad \text{for} \quad \forall k \in \mathbb{N}$$



S=5