Multichannel Image Restoration Based on Optimization of the Structural Similarity Index

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Motivation

- SPIM delievers images recorded from different angles
- These images need to be:
  - Registered
  - Fused to one single image
- Existing fusion algorithms:
  - Frequency based
  - Deconvolution based on optimization of the MSE
- New fusion algorithm:
  - Deconvolution based on optimization of SSIM

“An optimized system is only as good as the optimization criterion used to design it.”

DEFINITION

- MSE = Minimum Square Error

$$MSE(x,y)=\frac{1}{N}\sum_{i=1}^{N}(x_i-y_i)^2$$

- SSIM[1] = Structural Similarity Index Measure

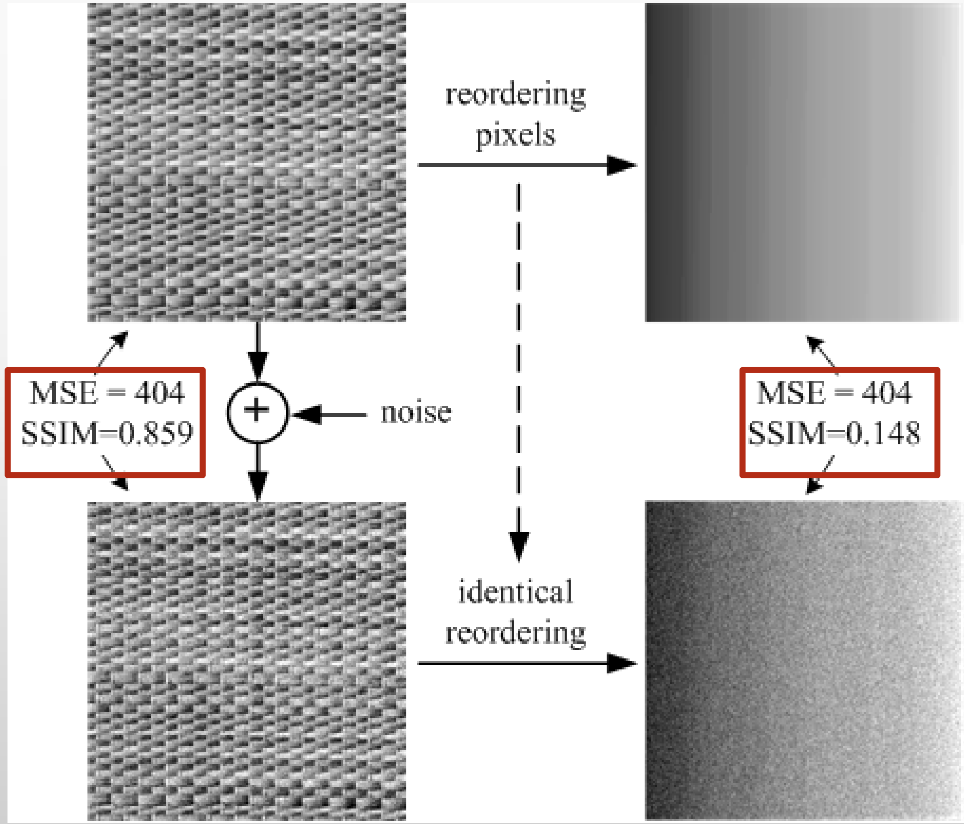
$$SSIM(x,y)=\underbrace{\frac{2\mu_x\mu_y+C_1}{\mu_x^2+\mu_y^2+C_1}}_{\textbf{luminance}}\cdot\underbrace{\frac{2\sigma_x\sigma_y+C_2}{\sigma_x^2+\sigma_y^2+C_2}}_{\textbf{contrast}}\cdot\underbrace{\frac{\sigma_{xy}+C_3}{\sigma_x\sigma_y+C_3}}_{\textbf{structure}}$$

[1]Z. Wang et al, IEEE TIP, vol. 13, no. 4, pp. 600-612, Apr. 2004

EXAMPLE[2]

- The structure of the image is important for the visual similarity!!

SSIM
High
(noise not visible
in the structure)



SSIM
Low
(noise disturbs the
smooth structure)

[2]Z. Wang and A.C. Bovik, IEEE Signal Processing Magazine, vol. 26, no. 1, 2009

EXAMPLE[2]

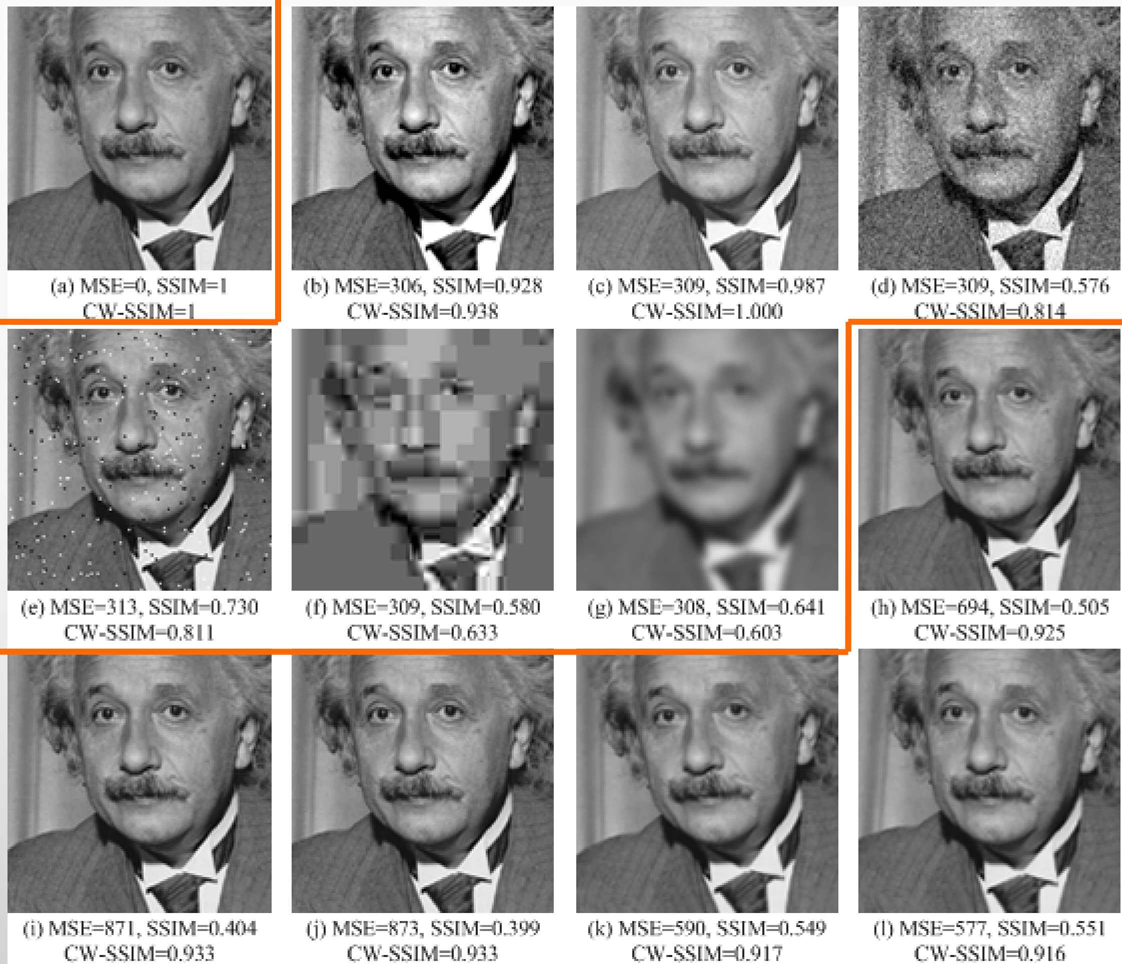
a) Reference image

b) Mean contrast stretch
c) Luminance shift

d) Gaussian noise
e) Implusive noise
f) JPEG compression
g) Blurring

h) Zooming out
i) Translation to right
j) Translation to left
k) Rotation counter-clockwise
l) Rotation clockwise

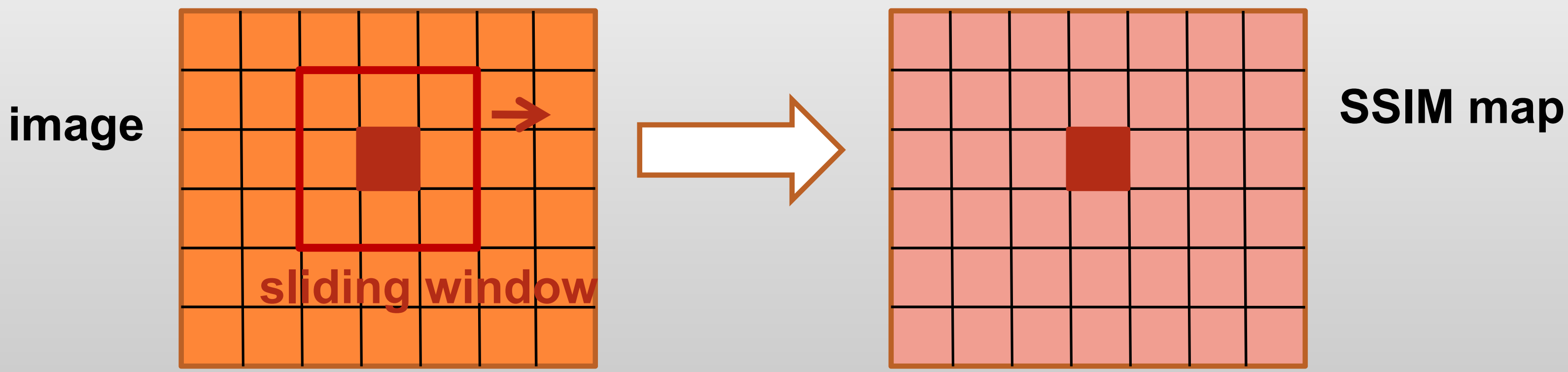
Almost identical MSE!!



[2]Z. Wang and A.C. Bovik, IEEE Signal Processing Magazine, vol. 26, no. 1, 2009

Computation of SSIM

- SSIM is computed locally within a sliding window that moves pixel by pixel across the image
- For each pixel the result is stored in a SSIM map
- The SSIM value of the whole image can be obtained by averaging the values from the SSIM map



Problem Outline

- The recorded image **y** can be described as a convolution of the original image **x** and the point spread function **h** plus the noise η introduced by the recording system:

$$y = h * x + \eta$$

- GOAL of multi-channel restauration:
  - ***Find the best estimate for*** x ***given the recorded images*** y_i
  - ***The quality of the estimate*** $\hat{x}$ ***is computed maximizing the structural similarity index measure***

PROBLEM OUTLINE II

<u>Basic Idea</u>: *Turn **non-convex** problem into a **quasi convex** problem*

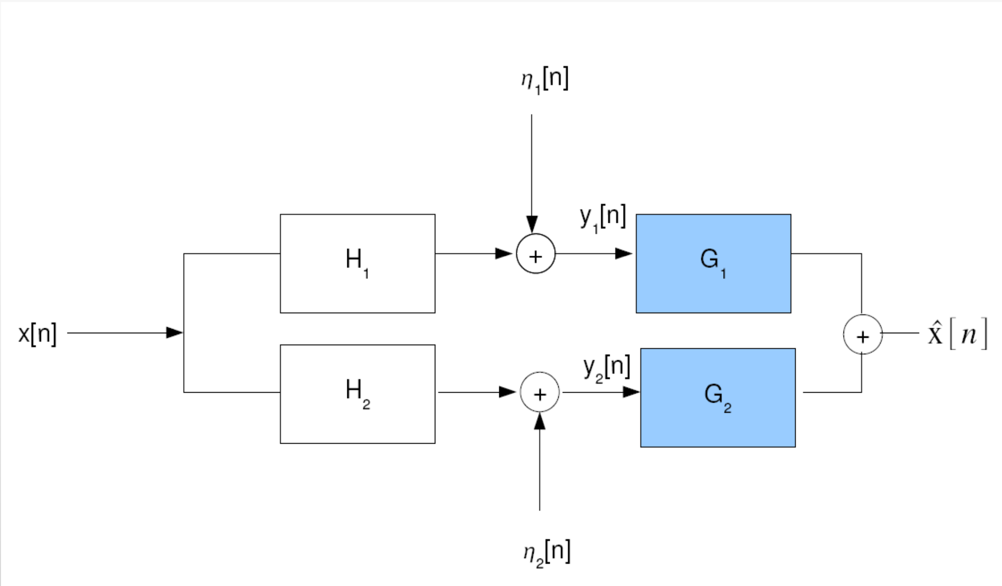
- We use the simplified SSIM[1]:

$$SSIM(x,\hat{x}) = \underbrace{\frac{2\mu_x\mu_{\hat{x}} + C_1}{\mu_x^2 + \mu_{\hat{x}}^2 + C_1}}_{Q_1} \cdot \underbrace{\frac{2\sigma_{x\hat{x}} + C_2}{\sigma_x^2 + \sigma_{\hat{x}}^2 + C_2}}_{Q_2}$$

- It is obtained from the original SSIM index by choosing $C_3 = C_2/2$

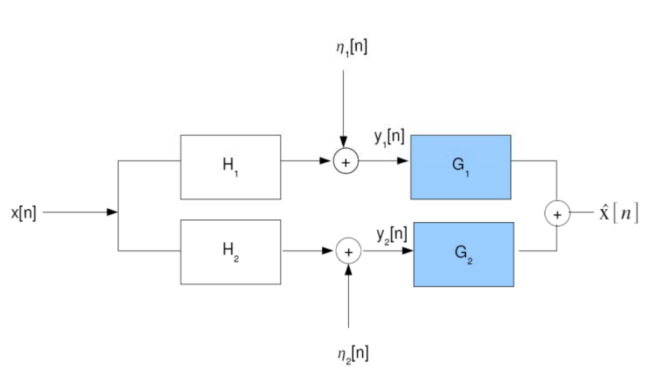
Extension Of Previous Work

- Restoration problem was solved for single channel images[4]
- We extend the solution to multi-channel images:



[4]S.S. Channappayya, IEEE TIP, vol. 17, no. 6, 2008

PROBLEM FORMULATION



- GIVEN:
  - Recorded image $\mathbf{y}_1, \ldots, \mathbf{y}_M$
  - Blurring filters $\mathbf{H}_1, \ldots, \mathbf{H}_M$
  - Probability density function of the noise

- GOAL:
  - Find inverse filters $\mathbf{g}_1, \ldots, \mathbf{g}_M$ such that:

$$\hat{x}[n] = g_1[n] * y_1[n] + \ldots + g_M[n] * y_M[n]$$

  - Maximizing the simplified SSIM

SOLUTION

$$SSIM(x,\hat{x}) = \underbrace{\frac{2\mu_x\mu_{\hat{x}} + C_1}{\mu_x^2 + \mu_{\hat{x}}^2 + C_1}}_{Q_1} \cdot \underbrace{\frac{2\sigma_{x\hat{x}} + C_2}{\sigma_x^2 + \sigma_{\hat{x}}^2 + C_2}}_{Q_2}$$

- Q_1 only depends on $g_i^T e$
- Constrain $g_i^T e$ to α_i
- The optimization problem is simplified to:

$$\hat{g}(\alpha) = \arg\max_{g \in \Re^{MN}} Q_2 \quad \text{subject to:} \quad g^T e = \alpha$$

- Where **g** is a matrix with the rows being the vectors $\mathbf{g}_1, \ldots, \mathbf{g}_\mathbf{M}$

SOLUTION II

- A boundary γ is set to obtain a quasi-complex optimization problem:

min: γ
subject to:
max: $Q_2 \leq \gamma$
subject to:
$$g^T e = \alpha$$

⟺

min: γ
subject to:
min: $f(\gamma) \geq 0$
subject to:
$$g^T e = \alpha$$

Lagrange Multipliers

- The overall problem is now convex and can be solved by applying the Lagrange multipliers

$$\nabla_{g_i}\left(f(\gamma)+\lambda_1\left(g_1^T e-\alpha_1\right)+\ldots+\lambda_M\left(g_M^T e-\alpha_M\right)\right)=0$$

$$\nabla_{\lambda_i}\left(f(\gamma)+\lambda_1\left(g_1^T e-\alpha_1\right)+\ldots+\lambda_M\left(g_M^T e-\alpha_M\right)\right)=0$$

- The optimal γ is computed using the bisection method

min: γ
subject to:
min: $f(\gamma)\geq 0$
subject to:
$g^T e=\alpha$

Solution For M = 2

$$\nabla_{\mathbf{g}_1}(f(\gamma)+\lambda_1(g_1^T e-\alpha_1)+\lambda_2(g_2^T e-\alpha_2)) = \gamma(2\mathbf{K}_{\mathbf{y}_1\mathbf{y}_1}\mathbf{g}_1+2\mathbf{K}_{\mathbf{y}_1\mathbf{y}_2}\mathbf{g}_2-2\mathbf{c}_{\mathbf{x}\mathbf{y}_1}+\lambda_1\mathbf{e})=0$$

$$\nabla_{\mathbf{g}_2}(f(\gamma)+\lambda_1(g_1^T e-\alpha_1)+\lambda_2(g_2^T e-\alpha_2)) = \gamma(2\mathbf{K}_{\mathbf{y}_2\mathbf{y}_1}\mathbf{g}_1+2\mathbf{K}_{\mathbf{y}_2\mathbf{y}_2}\mathbf{g}_2-2\mathbf{c}_{\mathbf{x}\mathbf{y}_2}+\lambda_2\mathbf{e})=0$$

$$\nabla_{\lambda_1}(f(\gamma)+\lambda_1(g_1^T e-\alpha_1)+\lambda_2(g_2^T e-\alpha_2)) = \mathbf{g}_1^{\mathbf{T}}\mathbf{e}-\alpha_1=0$$

$$\nabla_{\lambda_2}(f(\gamma)+\lambda_1(g_1^T e-\alpha_1)+\lambda_2(g_2^T e-\alpha_2)) = \mathbf{g}_2^{\mathbf{T}}\mathbf{e}-\alpha_2=0$$

$$\mathbf{g}_1=\mathbf{g}_{1,0}+\lambda_1\mathbf{g}_{1,1}+\lambda_2\mathbf{g}_{1,2}$$

$$\mathbf{g}_2=\mathbf{g}_{2,0}+\lambda_1\mathbf{g}_{2,1}+\lambda_2\mathbf{g}_{2,2}$$

$$\mathbf{g}_{1,0} := \frac{1}{\gamma}(\mathbf{K}_{\mathbf{y}_1\mathbf{y}_1}^{-1}(\mathbf{c}_{\mathbf{x}\mathbf{y}_1}-\mathbf{K}_{\mathbf{y}_1\mathbf{y}_2}\mathbf{K}^{-1}\mathbf{c}))$$

$$\mathbf{g}_{1,1} := -(\frac{1}{2\gamma}\mathbf{K}_{\mathbf{y}_1\mathbf{y}_1}^{-1}(\mathbf{I}-\mathbf{K}_{\mathbf{y}_1\mathbf{y}_2}\mathbf{K}^{-1}\mathbf{K}_{\mathbf{y}_2\mathbf{y}_1}\mathbf{K}_{\mathbf{y}_1\mathbf{y}_1}^{-1})\mathbf{e})$$

$$\mathbf{g}_{1,2} := \frac{1}{2\gamma}\mathbf{K}_{\mathbf{y}_1\mathbf{y}_1}^{-1}\mathbf{K}_{\mathbf{y}_1\mathbf{y}_2}\mathbf{K}^{-1}\mathbf{e}$$

$$\mathbf{g}_{2,0} := \frac{1}{\gamma}\mathbf{K}^{-1}\mathbf{c}$$

$$\mathbf{g}_{2,1} := \frac{1}{2\gamma}\mathbf{K}^{-1}\mathbf{K}_{\mathbf{y}_2\mathbf{y}_1}\mathbf{K}_{\mathbf{y}_1\mathbf{y}_1}^{-1}\mathbf{e}$$

$$\mathbf{g}_{2,2} := \frac{1}{2\gamma}\mathbf{K}^{-1}\mathbf{e},$$

$$\mathbf{K} := \mathbf{K}_{\mathbf{y}_2\mathbf{y}_2}-\mathbf{K}_{\mathbf{y}_2\mathbf{y}_1}\mathbf{K}_{\mathbf{y}_1\mathbf{y}_1}^{-1}\mathbf{K}_{\mathbf{y}_1\mathbf{y}_2}$$

$$\mathbf{c} := \mathbf{c}_{\mathbf{x}\mathbf{y}_2}-\mathbf{K}_{\mathbf{y}_2\mathbf{y}_1}\mathbf{K}_{\mathbf{y}_1\mathbf{y}_1}^{-1}\mathbf{c}_{\mathbf{x}\mathbf{y}_1}$$

IMPLEMENTATION

- Filter is implemented pixelwize for a neighborhood of size K×K (here K = 35)
- the covariance $\mathbf{c}_{\mathbf{xy}}$ is estimated using a heuristic technique described by Portilla and Simoncelli[5]
- Each block is made zero-mean before computing the inverse filter; the mean is added back after the computation
- Implementation in Matlab R2009a, for images of size **50×50 pixels** the computation time is **30 sec** on a Intel Core Duo processor with 3 GHz

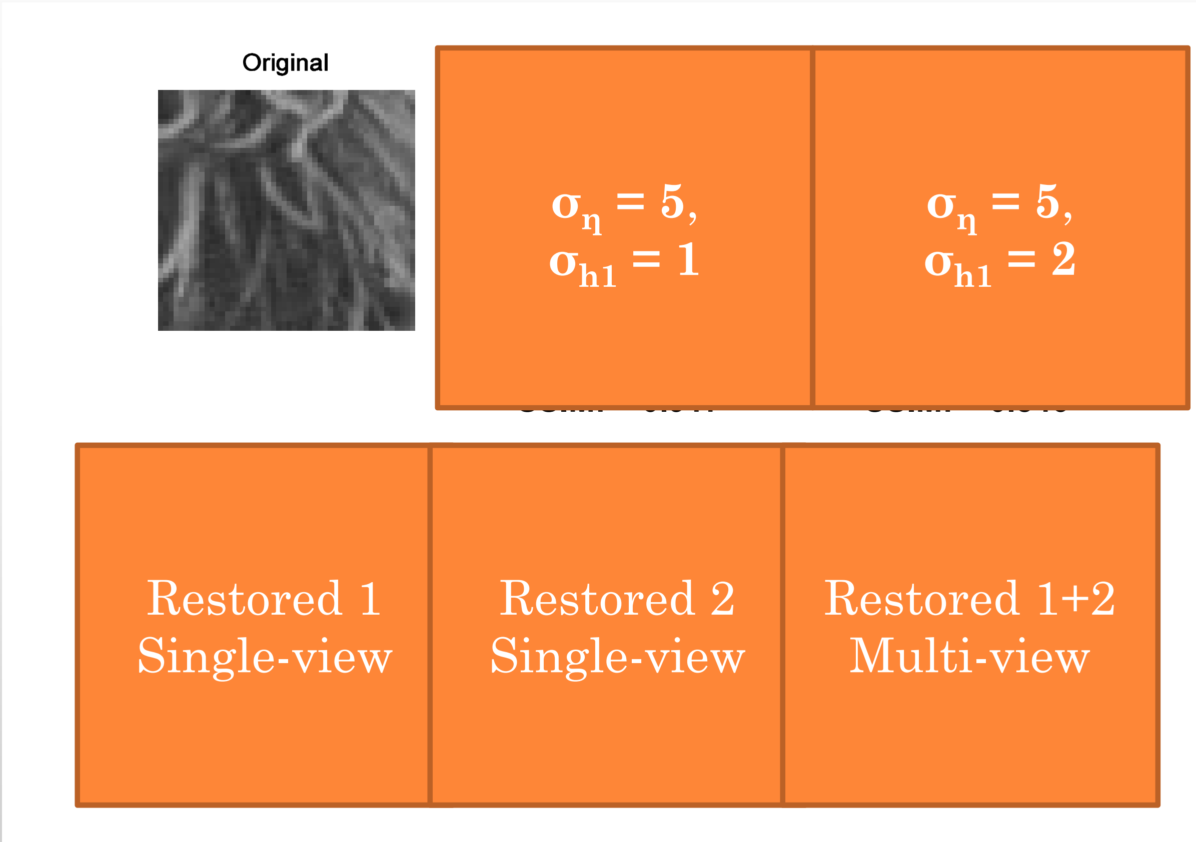
[5]J. Portilla and E. Simoncelli, Proc. of IEEE TIP, vol. 2, pp. 965-968, 2003

Results Lena

- Compare single vs. Multiview reconstruction

The original lena image (top row left) is distorted by $\boldsymbol{\sigma_\eta = 5}$, $\boldsymbol{\sigma_{h1} = 1}$ and $\boldsymbol{\sigma_{h2} = 2}$ resulting in the images Distorted 1 and Distorted 2 (top row middle and right).

After SSIM restoration is applied the results are presented for single-channel restoration of Distorted 1 and Distorted 2 (bottom row left and middle) and multi-channel restoration (bottom row right).

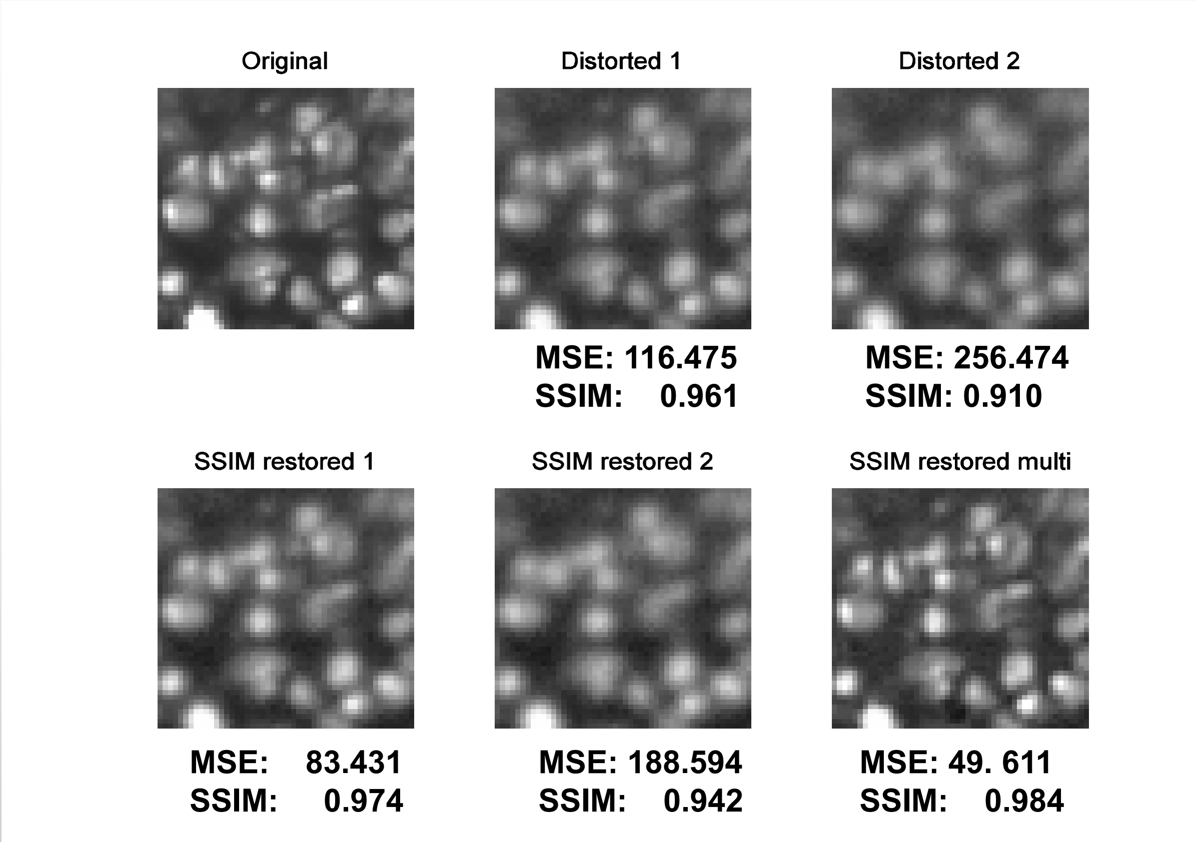


RESULTS DROSOPHILA

- Compare single vs. Multiview reconstruction

The original drosophila image (top row left) is distorted by $\sigma_\eta = 3$, $\sigma_{h1} = 1$ and $\sigma_{h2} = 2$ resulting in the images Distorted 1 and Distorted 2 (top row middle and right).

After SSIM restoration is applied the results are presented for single-channel restoration of Distorted 1 and Distorted 2 (bottom row left and middle) and multi-channel restoration (bottom row right).

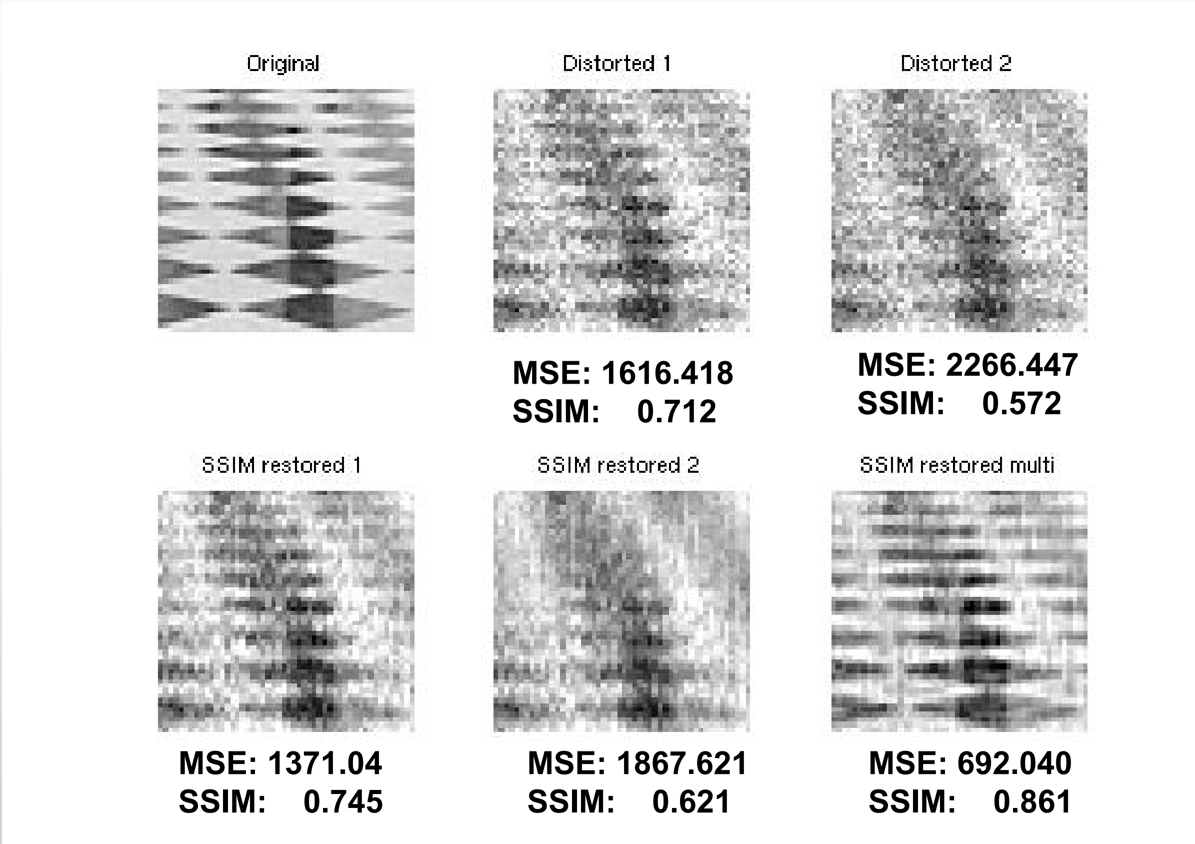


Results Checkboard

- Compare single vs. Multiview reconstruction

The original checkboard image (top row left) is distorted by $\boldsymbol{\sigma_\eta = 30}$, $\boldsymbol{\sigma_{h1} = 1}$ and $\boldsymbol{\sigma_{h2} = 2}$ resulting in the images Distorted 1 and Distorted 2 (top row middle and right).

After SSIM restoration is applied the results are presented for single-channel restoration of Distorted 1 and Distorted 2 (bottom row left and middle) and multi-channel restoration (bottom row right).

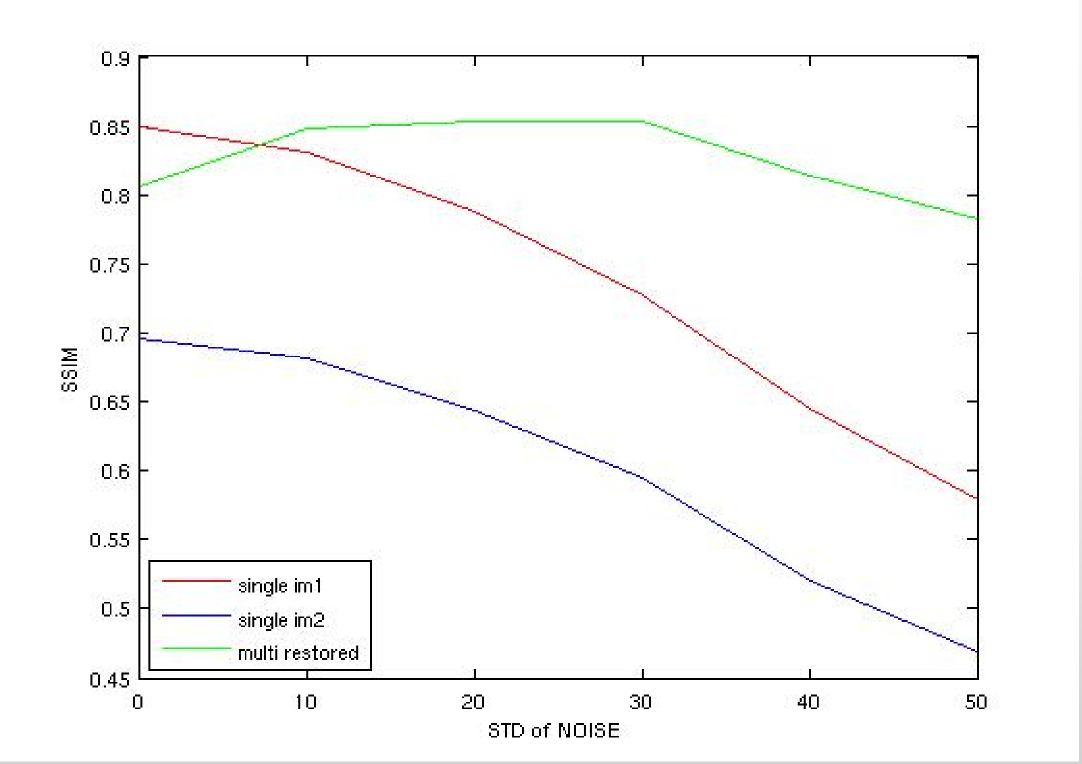


Results: Influence Of Noise

- Compare single vs. Multiview reconstruction while alternating the noise on chessboard image

The **influence of noise** (x-axis) on the **SSIM index** (y-axis) is plotted for single-channel restoration (red and blue) and multi-channel restoration (green).

σ_{h1} = 1 and **σ_{h2} = 2**



Conclusions

- Multi-channel SSIM image restoration significantly improves the single-channel SSIM restoration.

- Advantages of multi-channel SSIM restoration:

  - very effective if the noise level is high
  - a small filter size is sufficient to achieve optimal reconstruction results
  - local structures are preserved

Outlook

- Disadvantages of the method:
  - high computation time
  - needs an estimate for the original image

- Future research:
  - extension to blind deconvolution
  - application to three dimensional images
  - significance of the number of distorted images M

Project Team

- Computer Science:
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  - Olaf Ronneberger
  - Maja Temerinac-Ott
  - Dominik Rueß
  - Mario Emmenlauer
- Biology I:
  - Wolfgang Driever
  - Alida Filippi
  - Björn Wendik
- ZBSA
  - Roland Nitschke

Thank you for your attention!

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Backup

MSE vs SSIM

MSE

- Fast & easy to compute
- Valid distance metric in $\mathbf{R}^N$
- Natural way to define energy of error signal
- Convex, symmetric and differentiable
- Widely used

SSIM

- Models similarity as perceived by human visual system
- Natural image signals are highly structured (strong neighborhood dependencies)
- Symmetric, bounded and has a unique maximum

DEFINTION II

- mean intensity: $$\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i$$
- Standard deviation: (signal contrast) $$\sigma_x = \left(\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu_x)^2\right)^{\frac{1}{2}}$$
- Covariance: $$\sigma_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu_x)(y_i - \mu_y)$$
- stabalizing constants: C_1, C_2, C_3

DISTORTIONS

Structural distortions

- Additive noise and blur
- Lossy compression

➡ SSIM describes the visual quality good

Non-structural distortions

- Change of luminance and brightness
- Change of contrast
- Gamma distortion
- Spatial shift

➡ Better use Complex Wavelet SSIM[3]

[3] Z. Wang and E. P. Simoncelli, Trans. IEEE ICASSP, vol. 2, pp. 573-576, 2005

PROBLEM FORMULATION II

- The inverse filters $\mathbf{g_1}, \ldots, \mathbf{g_M}$ are found adjointly by optimizing the statistical SSIM index[3]

$$\hat{g} = \arg\max_{g \in \Re^{MN}} StatSSIM\left(x[n], \hat{x}[n]\right)$$

- Where:

$$StatSSIM\left(x[n], \hat{x}[n]\right) = \underbrace{\frac{2\mu_x \mu_{\hat{x}} + C_1}{\mu_x^2 + \mu_{\hat{x}}^2 + C_1}}_{Q_1} \cdot \underbrace{\frac{2\sigma_{x\hat{x}} + C_2}{\sigma_x^2 + \sigma_{\hat{x}}^2 + C_2}}_{Q_2}$$

Statistical SSIM Index

$$\mu_x = E[x[n]]$$

$$\sigma_x^2 = E\left[(x[n] - \mu_x)^2\right]$$

$$\sigma_{xy} = E[(x[n] - \mu_x)(y[n] - \mu_y)]$$

BISECTION METHOD

- The optimal γ is computed using the bisection method

1. Initialize γ (say γ_0) between 0 and 1.
Set $upLimit = 1$, $lowLimit = \gamma_0$
2. Evaluate the optimal filter.
if $f(\gamma) \geq 0$ **then**
  if $(upLimit - lowLimit) < \epsilon$ **then**
    Optimal γ found.
    Exit.
  else
    Set $\gamma = (upLimit - lowLimit)/2$
    $upLimit = \gamma$.
    Go to step **2**.
  end if
else
  Set $\gamma = (upLimit - lowLimit)/2$
  $lowLimit = \gamma$.
  Go to step **2**.
end if

EXPLANATION

$\mathbf{K_{yy}}$ - covariance matrix
$\mathbf{c_{xy}}$ – cross covaraince vector

- Explicitly Q_2 is:

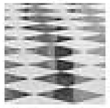
$$\mathbf{Q_2} = \frac{2E[(\mathbf{x}[n]-\mu_{\mathbf{x}})(\sum_{k=1}^{M}\sum_{i=0}^{N-1}(\mathbf{g_k}[i]-\mu_{\mathbf{y_k}})]+C_2}{E[(\mathbf{x}[n]-\mu_{\mathbf{x}})^2]+E[(\sum_{k=1}^{M}\sum_{i=0}^{N-1}\mathbf{g_k}[i]-\mu_{\mathbf{y_k}})^2]+C_2}$$

$$= \frac{2\sum_{i=1}^{M}\mathbf{g_i}^{\mathbf{T}}\mathbf{c_{xy_1}}+C_2}{\sigma_{\mathbf{x}}^2+2\sum_{i=1}^{M}\sum_{j=1}^{M}\mathbf{g_i}^{\mathbf{T}}\mathbf{K_{y_1y_j}}\mathbf{g_j}+C_2}$$

- And $f(\gamma)$ is:

$$f(\gamma) = \gamma(\sigma_{\mathbf{x}}^2+2\sum_{i=1}^{M}\sum_{j=1}^{M}\mathbf{g_i}^{\mathbf{T}}\mathbf{K_{y_1y_j}}\mathbf{g_j}+C_2) - (2\sum_{i=1}^{M}\mathbf{g_i}^{\mathbf{T}}\mathbf{c_{xy_1}}+C_2)$$

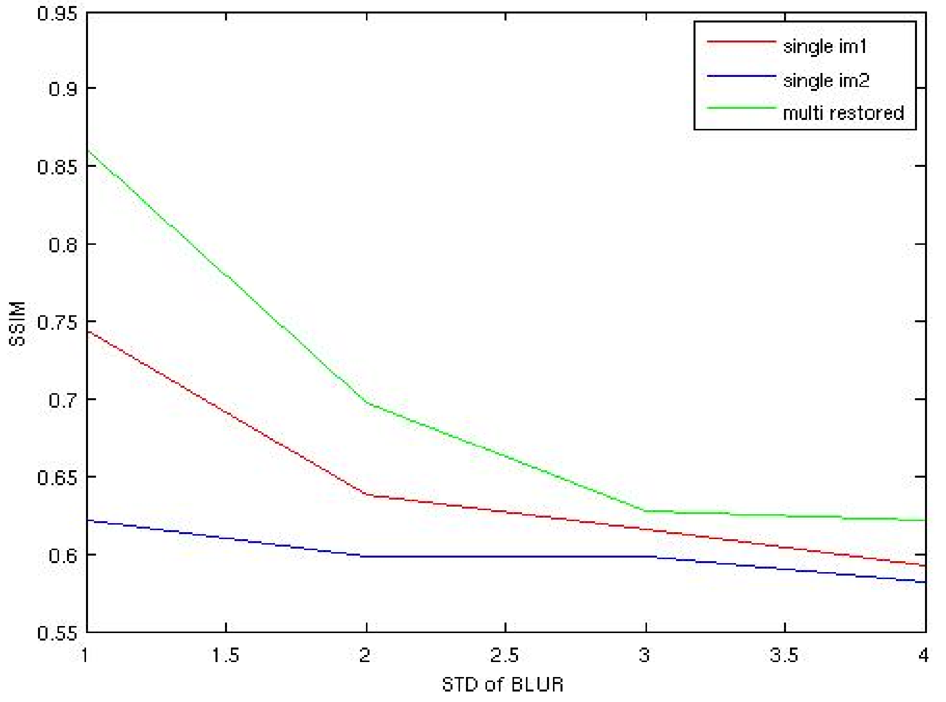
Original



RESULTS: INFLUENCE OF BLUR STD

- Compare single vs. Multiview reconstruction while alternating the blur STD for the checkboard image

The **SSIM values** of the single-channel restored image and the multi-channel restored image (y-axis) are plotted against the **blur size** $\boldsymbol{\sigma_{h1}}$ (x-axis)



Backup 9

RESULTS: INVERSE FILTER SIZE

- Compare single vs. Multiview reconstruction while alternating the filter size

The **SSIM values** of the single-channel restored image and the multi-channel restored image (y-axis) are plotted against the **filter size** (x-axis).

Here results for the lena image with parameters $\boldsymbol{\sigma_{h1} = 3}$, $\boldsymbol{\sigma_{h2} = 6}$ and $\boldsymbol{\sigma_{\eta} = 15}$ is presented.