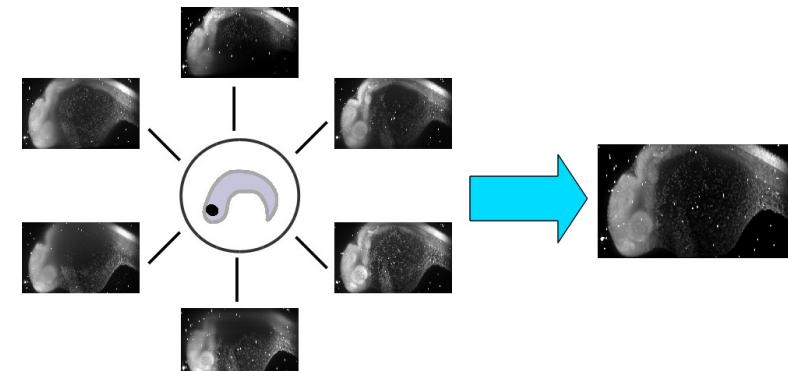


# SPATIALLY-VARIANT LUCY-RICHARDSON DECONVOLUTION FOR MULTIVIEW FUSION OF MICROSCOPICAL 3D IMAGES

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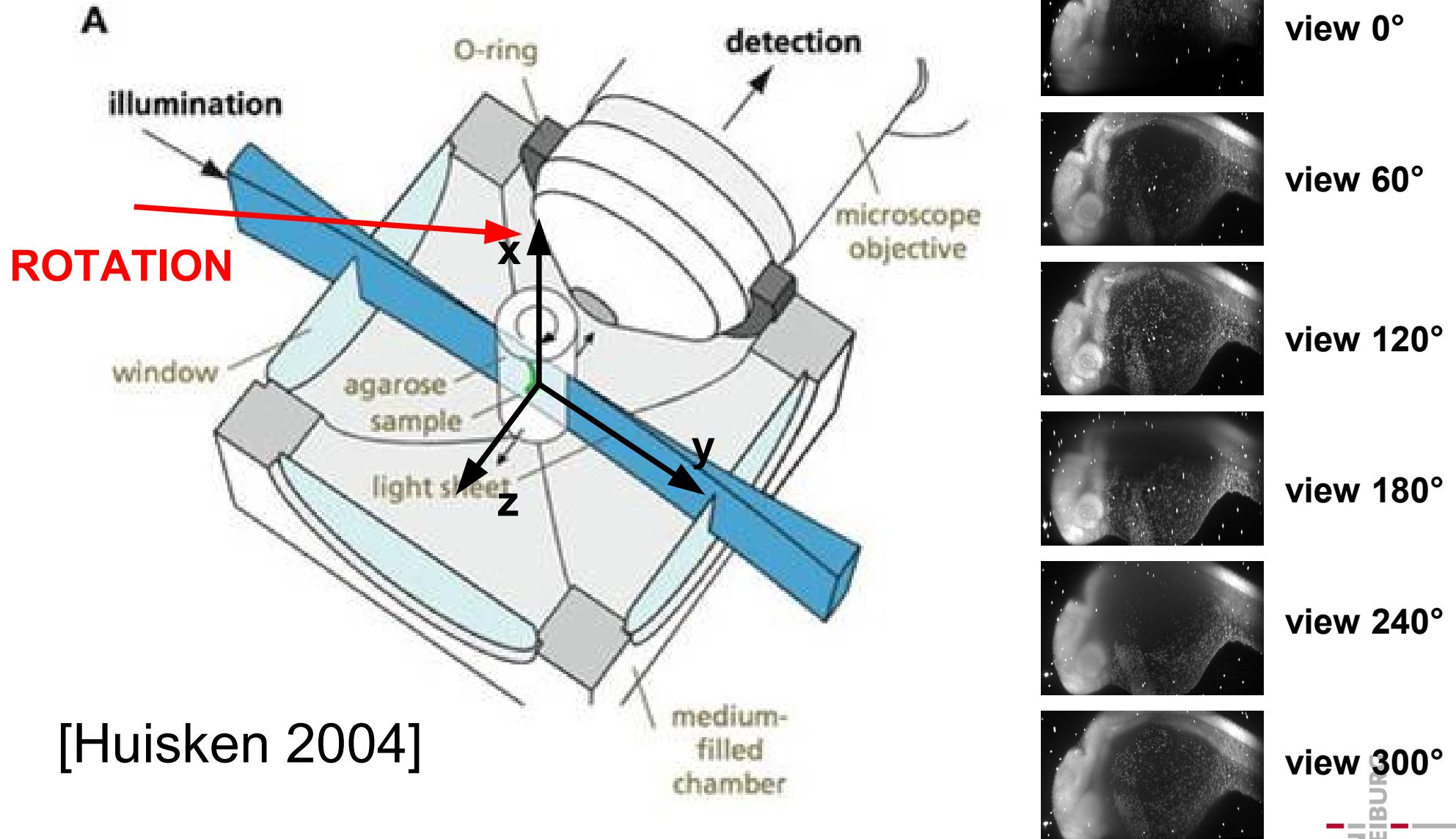
UNI  
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University of Freiburg, Germany

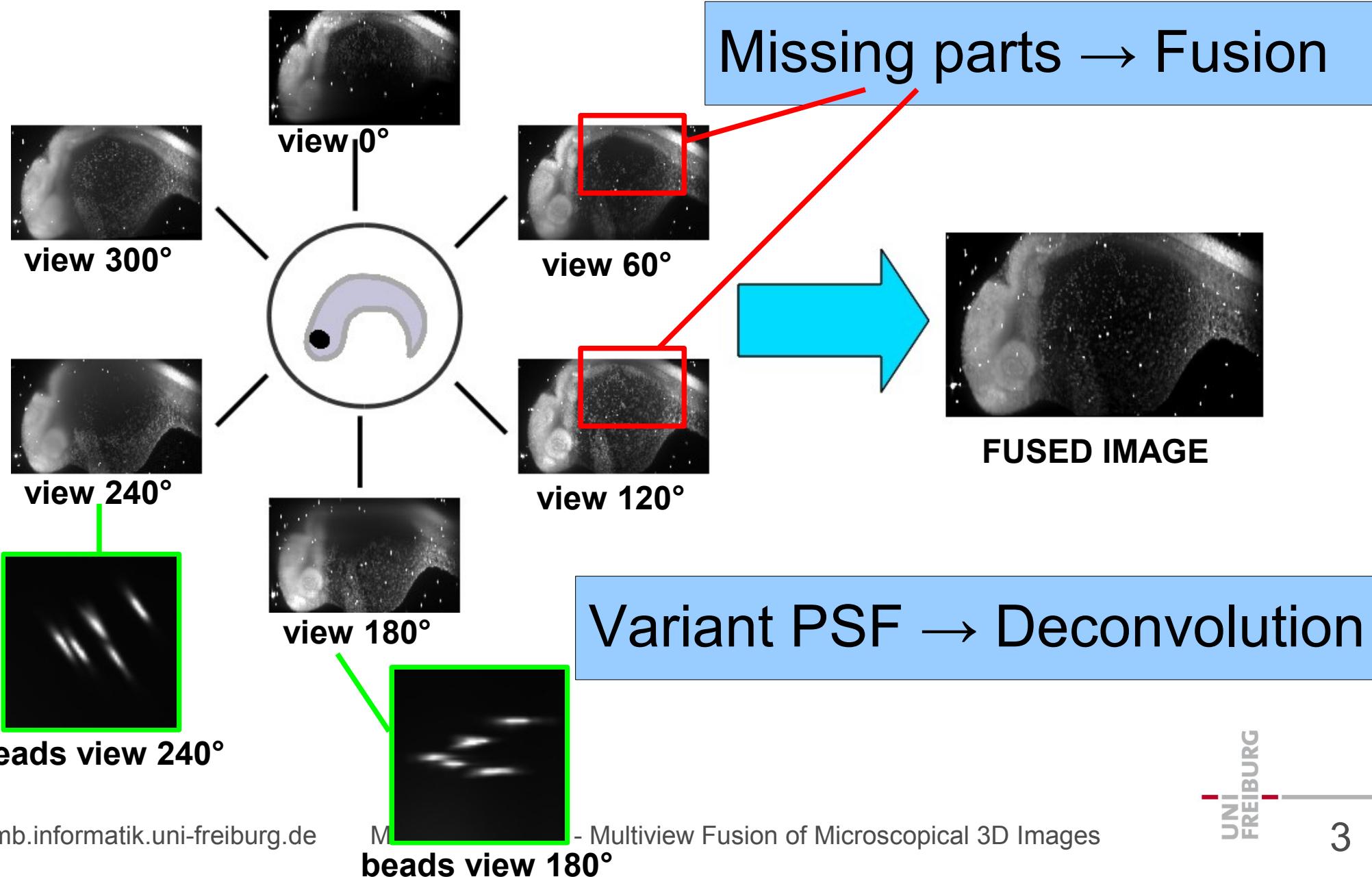


The Third LSM Workshop, Toulouse, October 13<sup>th</sup>-14<sup>th</sup>, 2011

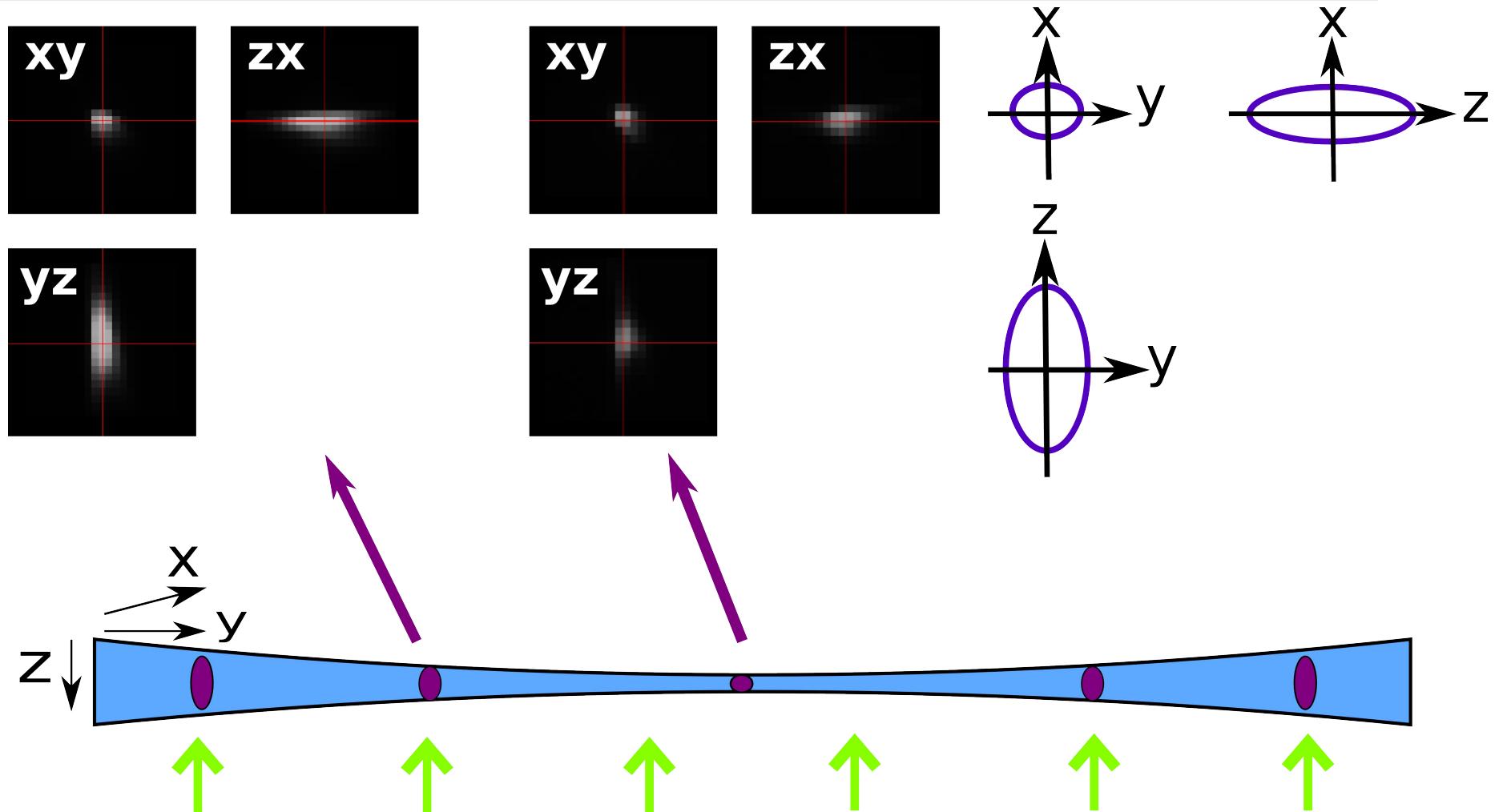
# SPIM = Single Plane Illumination Microscopy



# Goal: Joint Fusion and Deconvolution



# Estimation of the PSF at Bead Positions

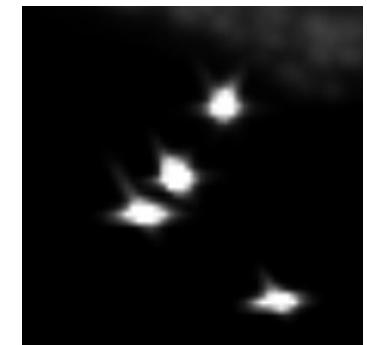
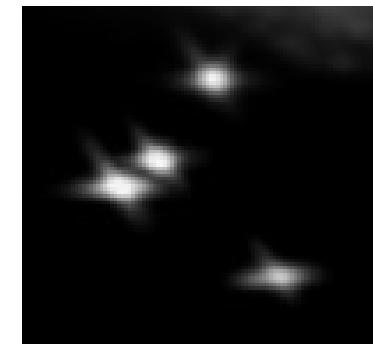


- Variation of the PSF along y-axis

# Related Work

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- Blending [Preibisch 2010]
  - Combines Gray values without a prior model
    - Fast Computation
    - Smearing of the points + blur
- Average PSF for Multiview Deconvolution [Krzic 2009]
  - Assumes constant PSF
    - Good in the center
    - Bad at the corners of the image



# Contribution

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- Location **variant PSF estimation** for joint deconvolution and fusion
- Approach:
  - PSF Estimation
  - Overlap-Save Deconvolution
  - Lucy-Richardson Algorithm
  - Multiview deconvolution
  - TV Regularization



**LRMOS-TV**

# Problem Formulation: Multiview Fusion

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➤ **Given:**

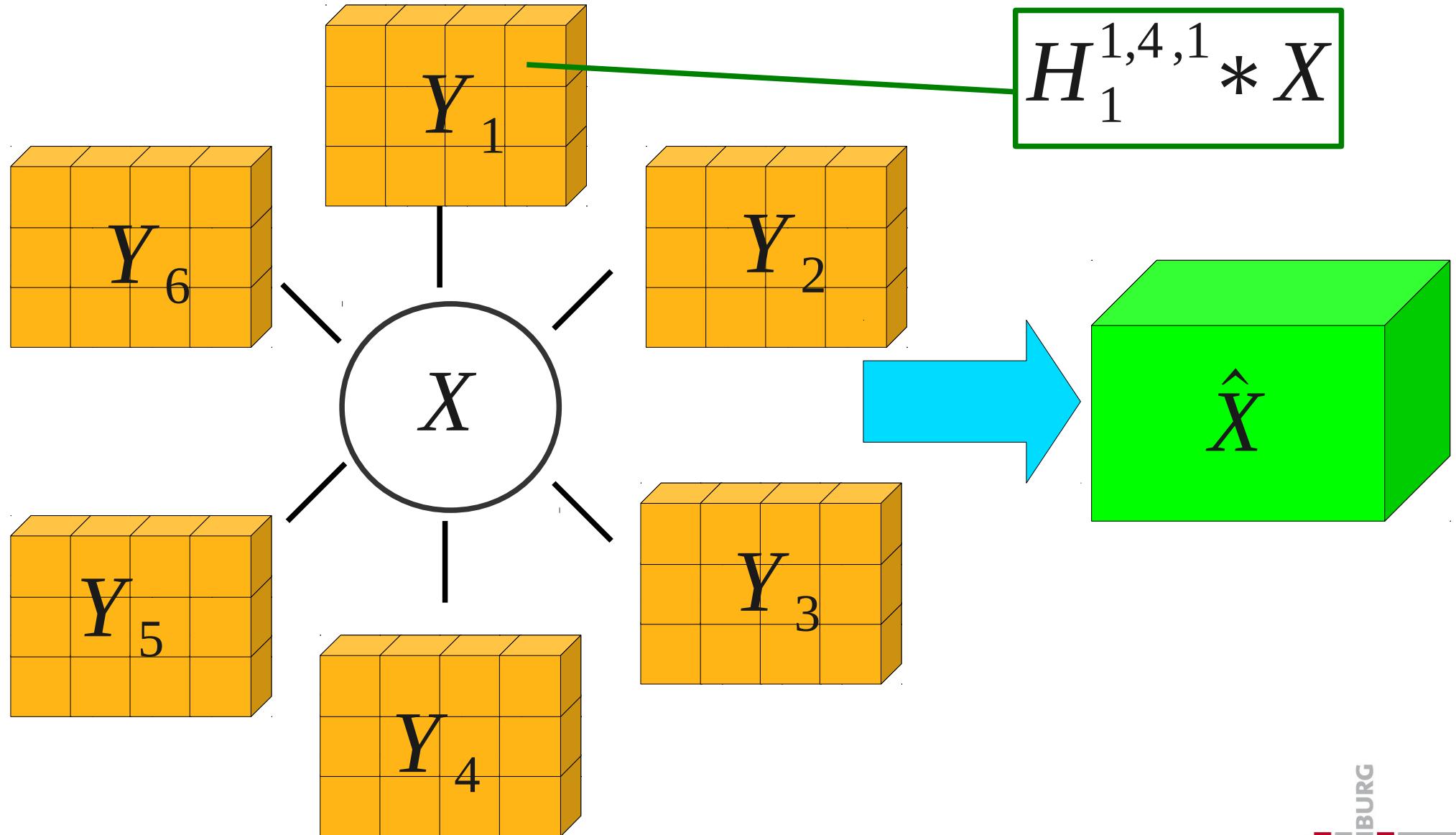
- **Recorded images**  $Y_1, \dots, Y_N$
- **PSF at bead positions**  $H_1, \dots, H_N$

➤ **Goal:**

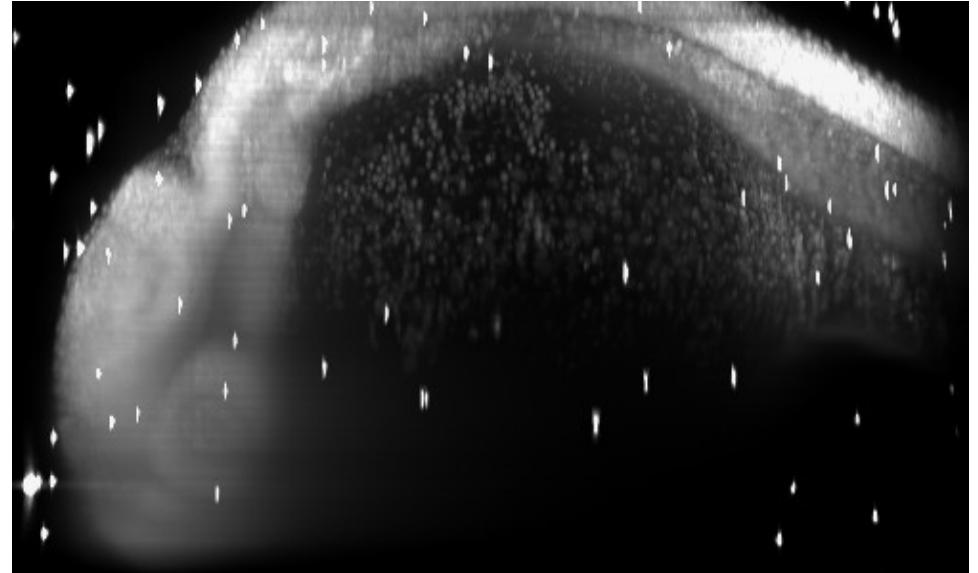
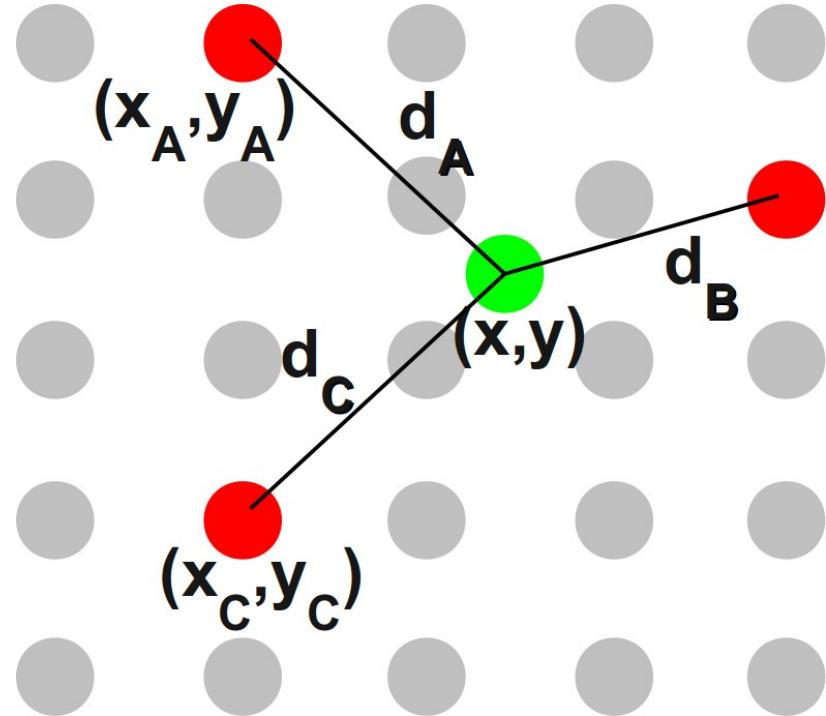
- **Find true image**  $X$   
that maximizes

$$p(X|Y_1, \dots, Y_N, H_1, \dots, H_N) = \prod_{i=1}^N p(X|Y_i)$$

# Solution: Regionwise Multiview Fusion



# PSF Estimation



$$H(x, y) = \frac{d_B d_C H_A + d_A d_C H_B + d_A d_B H_C}{d_B d_C + d_A d_C + d_A d_B}$$

$$d_A = \sqrt{(x - x_A)^2 + (y - y_A)^2}$$

# Overlap-Save Deconvolution

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- Model spatially-variant PSF by blockwise constant PSFs
- Consider large overlapping regions to overcome boundary artifacts

$$\begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \dots & Y_{nn} \end{bmatrix} \xrightarrow{\text{green arrow}} Y_{ij}^{(r+s)} = \begin{bmatrix} \times & \times & \times \\ \times & Y_{ij} & \times \\ \times & \times & \times \end{bmatrix}$$

Size of the blocks:  
 $s \times s$

Size of the padded blocks:  
 $(s+r) \times (s+r)$

# Image Formation Model

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- Convolution with the PSF of the system:

$$Y = P(H * X)$$

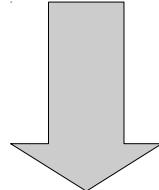
The diagram illustrates the Image Formation Model. At the top, the equation  $Y = P(H * X)$  is displayed. Below it, four components are arranged horizontally: "Recorded Image" on the left, "PSF" in the center, "True" Image on the right, and "Noise Model" at the bottom. Green lines connect the "Recorded Image" to both the "True" Image and the "Noise Model". A vertical green line connects the "PSF" to the "True" Image.

# Deconvolution: MLE Estimation

- Image Statistics Modeled by Poisson Process [Herbert 1989]:

$$p(X|Y) = \prod_{\nu} \frac{[(H * X)(\nu)]^{Y(\nu)}}{\exp(-(H * X)(\nu))}$$

Likelihood Probability



$$J(X) = \int_{\nu} Y(\nu) \log[(H * X)(\nu)] - (H * X)(\nu) d\nu$$

log likelihood

# Lucy-Richardson Algorithm

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$$\hat{X}^{p+1}(\nu) = \hat{X}^p(\nu) \cdot C^p(\nu)$$



Correction Factor:

$$C^p(\nu) = (H^s * \frac{Y}{S^p})(\nu)$$

$$H^s(\nu) = H(-\nu)$$

Simulated Image

$$S^p = (H * \hat{X}^p)(\nu)$$

# Multiview Deconvolution

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- Total Correction Factor (CF) as average of the individual correction factors [Krzic 2009]:

$$C^p = \frac{1}{N} \sum_{i=1}^N C_i^p$$

- Computation of the individual CF:

$$C_i^p(\nu) = (H_i^s * \frac{Y_i}{S_i^p})(\nu)$$

$$S_i^p = (H_i * \hat{X}^p)(\nu)$$

# TV Regularization

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- › Regularization of the initial energy by Total Variation [Dey 2004] :

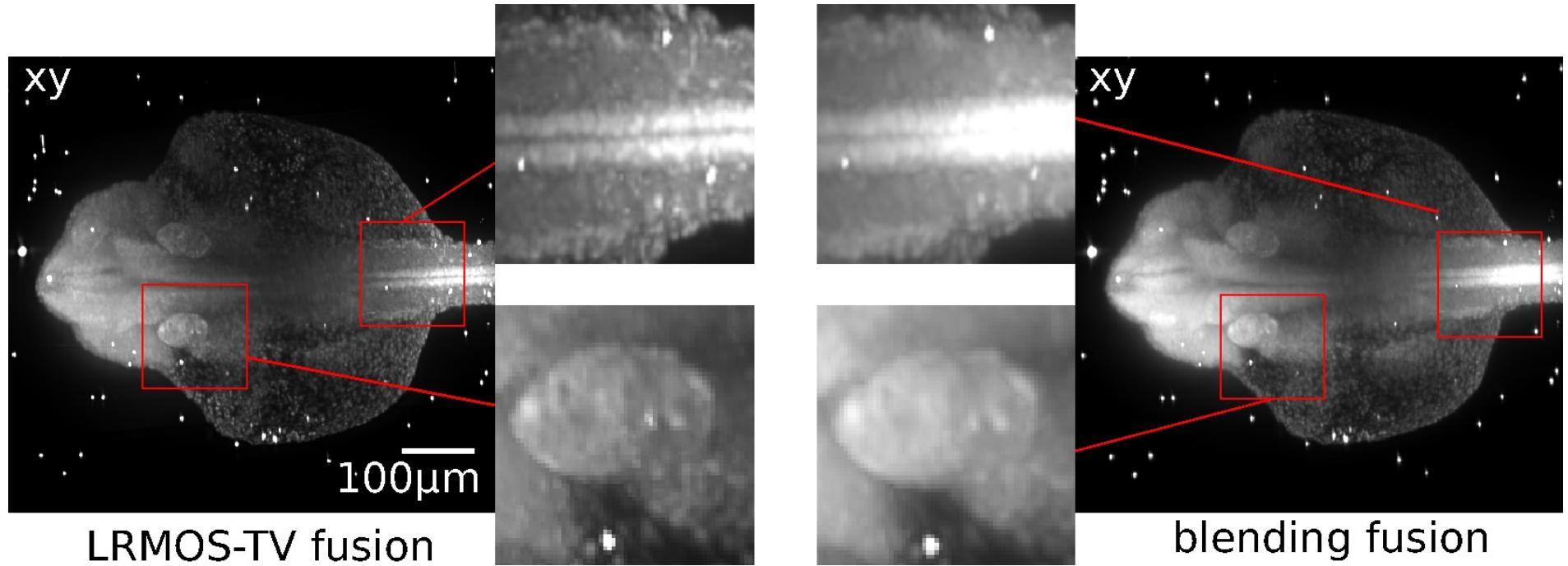
$$J_{TV}(X) = J(X) + \lambda \int_v |\nabla X(v)| d v$$

- › Resulting iteration using Green's one-step-late (OSL) algorithm:

$$\hat{X}^{p+1}(v) = \frac{\hat{X}^p(v)}{1 - \lambda \operatorname{div}\left(\frac{\nabla(\hat{X})^p(v)}{|\nabla(\hat{X})^p(v)|}\right)} \cdot C^p(v)$$

# Results: Visual Comparison to Blending

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Parameters:

$r = 11$ ,  $p = 4$ ,

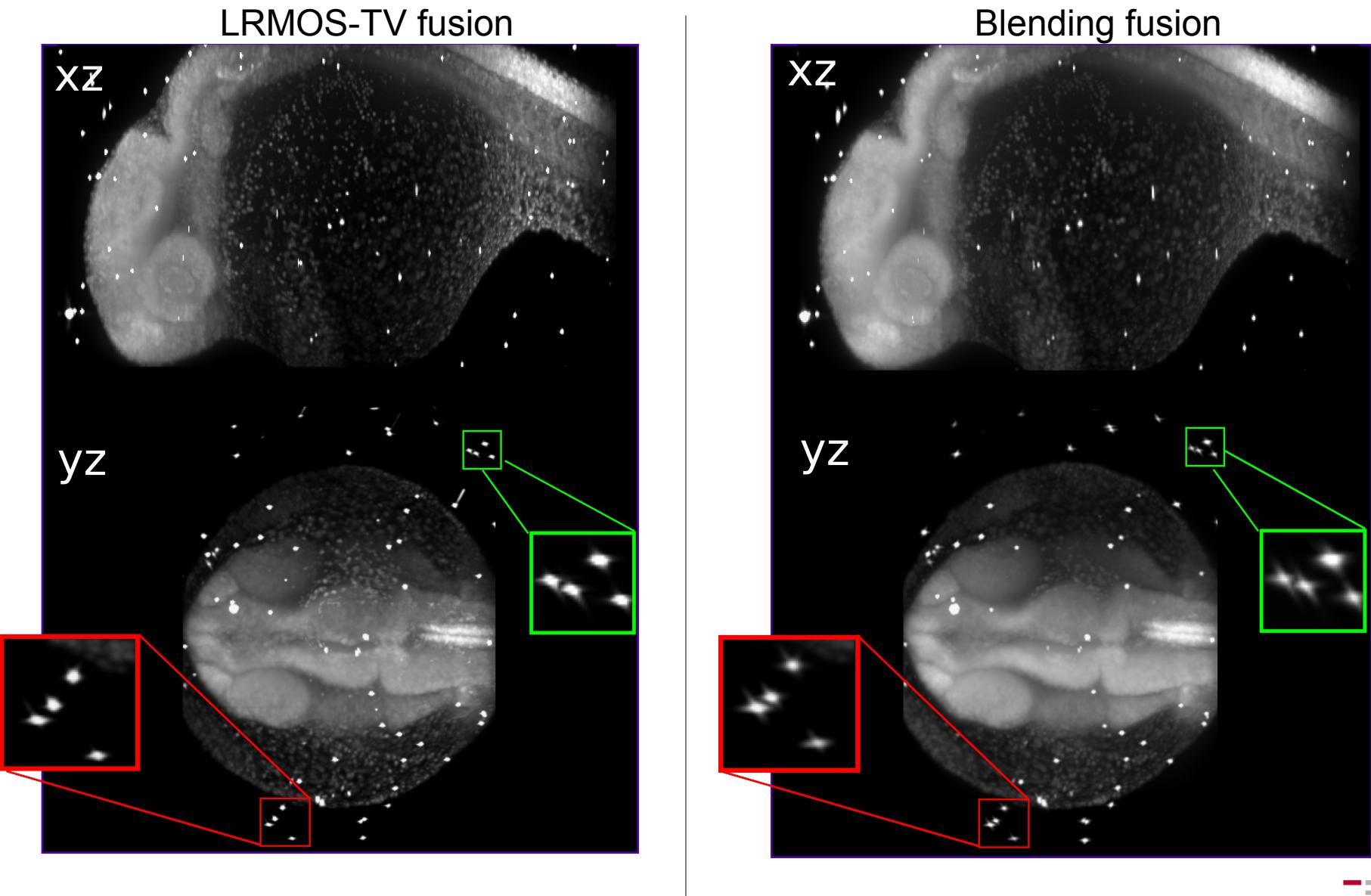
$s+r = 64$

Computation Time: 40 min

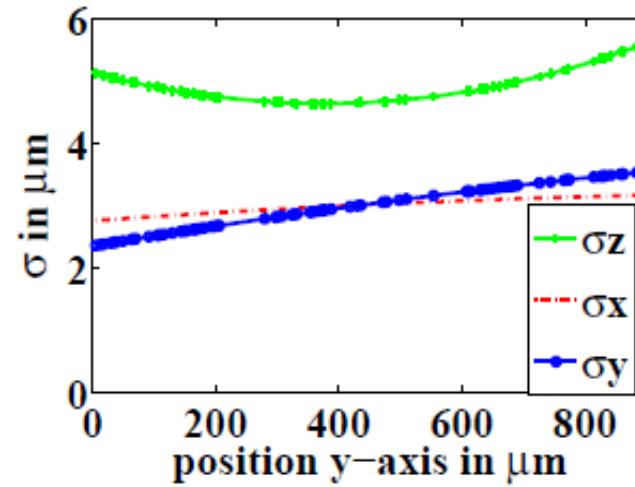
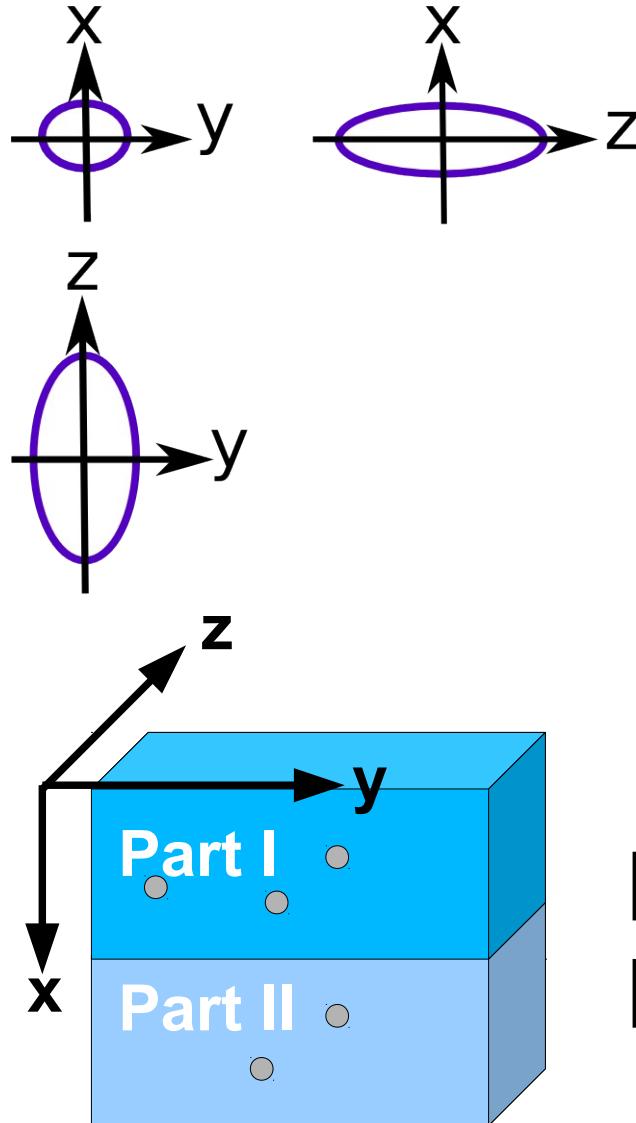
[Preibisch 2010]

Computation Time: 20 min

# Results: Visual Comparison to Blending



# Quantitative Evaluation

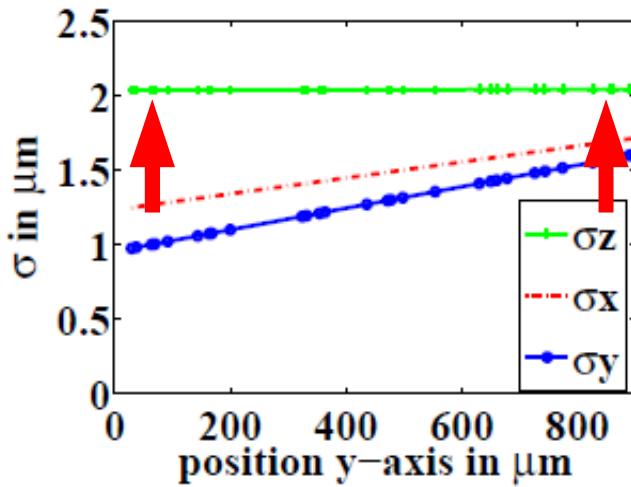
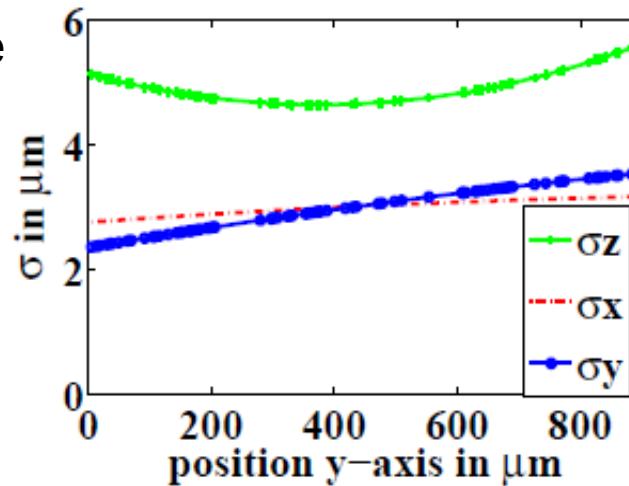


Original bead shape  
(single view)

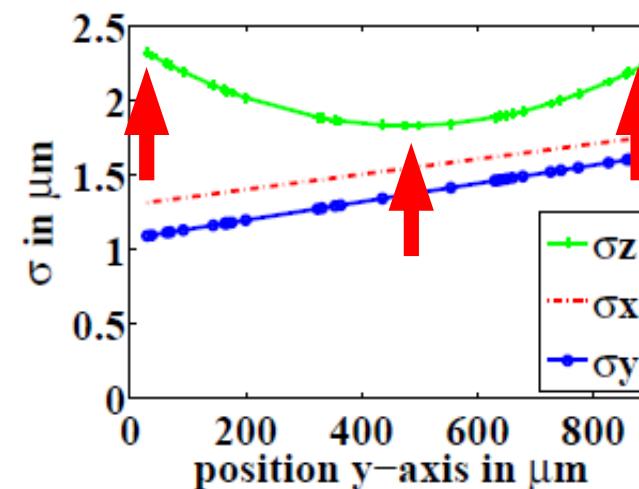
Part I: PSF Estimation  
Part II: Multiview Fusion

# Results: Comparison to Average PSF

Original bead shape  
(single view)

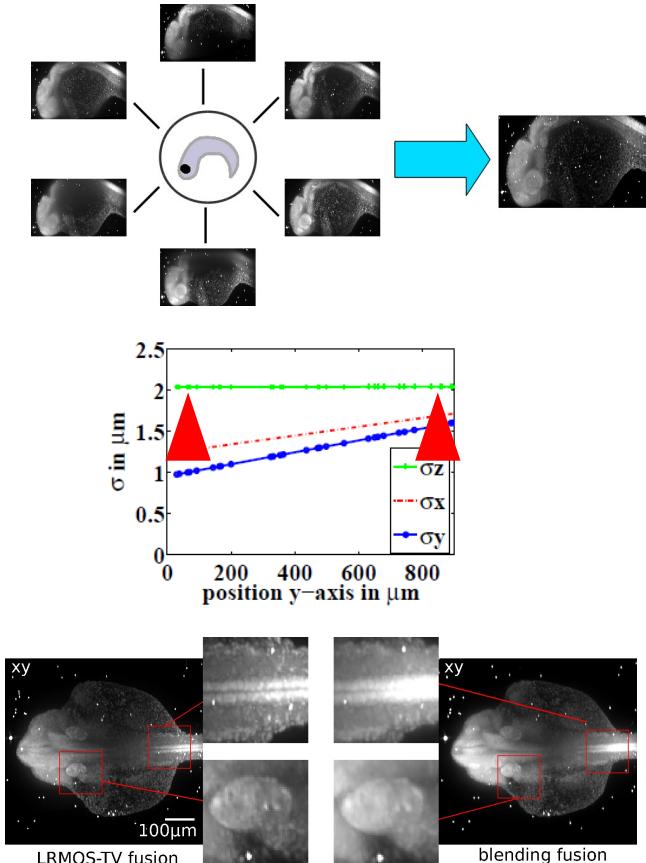


Deconvolved with variant PSF  
(fused image, upper)



Deconvolved with average PSF  
(fused image, upper)

# Conclusions



- A new framework for the fusion of the SPIM images was presented
- Spatially-variant Deconvolution better models the optical properties of the system than existing methods
- The structure borders are well preserved due to the TV regularization
- The algorithm is fast and can be easily parallelized

$$\begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \dots & Y_{nn} \end{bmatrix} \xrightarrow{\text{algorithm}} Y_{ij}^{(r+s)} = \begin{bmatrix} \times & \times & \times \\ \times & Y_{ij} & \times \\ \times & \times & \times \end{bmatrix}$$

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# Thank you for your attention!

# Proposed Algorithm ("LRMOS-TV")

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```
for  $m = 1$  to  $T_1$  do
    for  $n = 1$  to  $T_2$  do
        for  $k = 1$  to  $T_3$  do
            1. Extract extended region  $R_i = Y_{mnk}^{(r+s)}$ 
               from  $Y_i$  for each view  $i$ .
            2. Obtain  $H_i = H_{m,n,k}^{(r+s)}$ 
               by padding with zeros for each view  $i$ .
            3. Compute the initial estimate:
                 $\hat{X}^0 = \frac{1}{N} \sum_{i=1}^N R_i^p$ 
            4. Iterate:
                
$$\hat{X}_{m,n,k}^{p+1}(\mathbf{v}) = \frac{\hat{X}_{m,n,k}^p(\mathbf{v})}{1 - \lambda \text{div} \left( \frac{\nabla \hat{X}_{m,n,k}^p(\mathbf{v})}{|\nabla \hat{X}_{m,n,k}^p(\mathbf{v})|} \right)} \cdot C^p(\mathbf{v})$$

            5. Extract  $\hat{X}_{mnk}$  from  $\hat{X}_{mnk}^{(r+s)}$  and save into  $\hat{X}$ .
        end for
    end for
end for
```

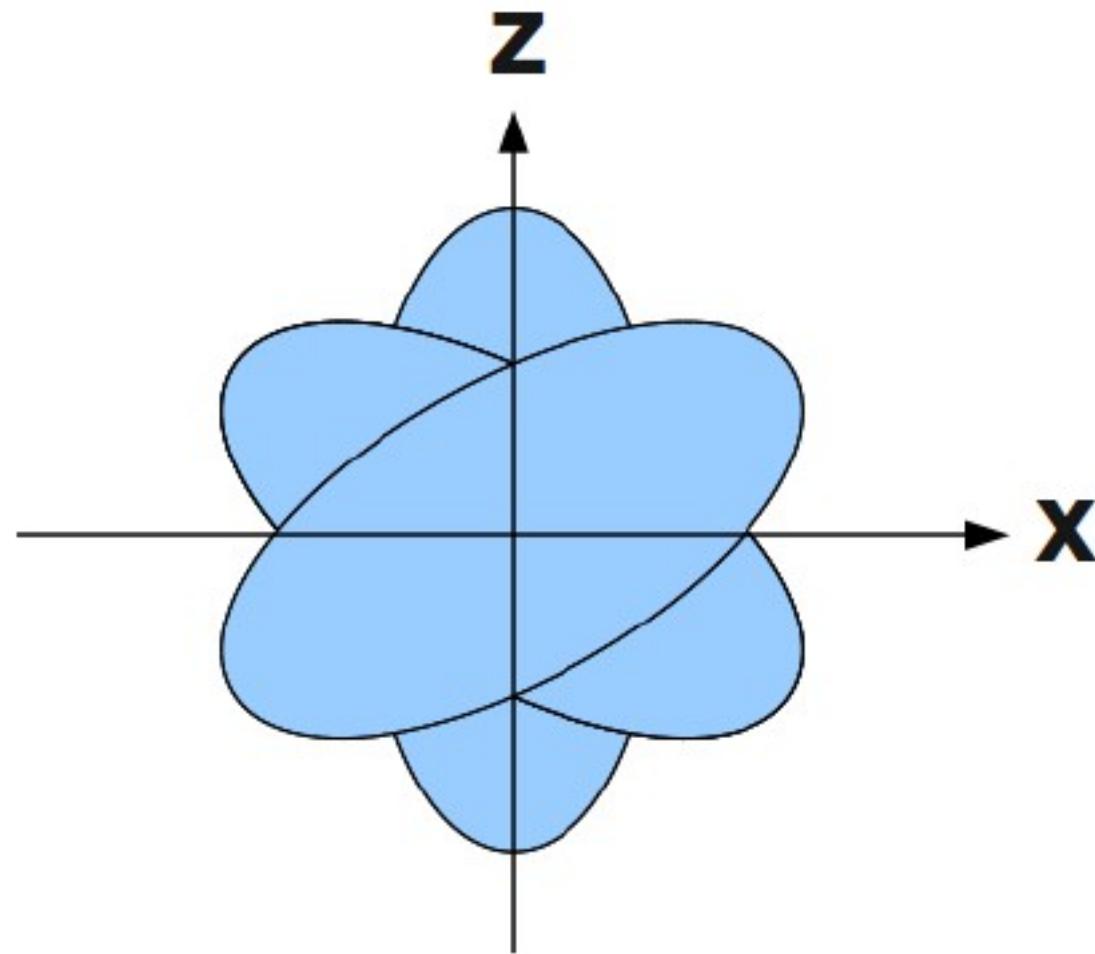
# Outlook

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- Algorithm:
  - Additional regularization strategies
  - Optimal number of iteration steps
  - A parametric model of the PSF along the lightsheet
- Microscopy:
  - Insert and record beads inside the sample for better PSF modeling inside the tissue
  - Automatic centering of the sample

# Coverage of the Beads in xz

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# Average PSF vs Variant PSF

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|            | deconvolved with<br>average PSF | deconvolved with<br>variant PSF |
|------------|---------------------------------|---------------------------------|
| $\sigma_x$ | 1.5375 (max 1.6409)             | 1.4835 (max 1.6179)             |
| $\sigma_y$ | 1.3598 (max 1.7707)             | 1.2937 (max 1.7233)             |
| $\sigma_z$ | 2.0252 (max 2.4188)             | 2.0354 (max 2.038)              |